CC3D Reference Manual

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Maciej H. Swat, Julio Belmonte, James A. Glazier

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Authors

The focus of this manual is to teach you how to use CC3DML (XML-based syntax) to build powerful multi-scale multi-cell tissue simulations. We will assume that you have a working knowledge of XML. You do not have to be an XML guru but you should know how to write simple XML documents

Authors

- Maciej H. Swat
- Julio Belmonte
- T.J. Sego
- James Glazier

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Introduction to CompuCell3D Modules CC3DML Syntax

This CompuCell3D reference material provides users with fairly detailed description of CC3DML syntax (CC3DML is XML-based model description format) of CC3D modules. The presented material is intended for users who are already familiar with CompuCell3D basics and know how to build and run simple simulations such as cell-sorting, bacterium macrophage or cell-type-oscillator. Since we often show CC3DML syntax and accompanying Python syntax for CC3D scripting we assume that users are familiar with Python. For readers who are using CompuCell3D for the first time we strongly recommend reading "Introduction To CompuCell3D". Complete description of CC3D Python scripting can be found in "CC3D Python Scripting Manual". Both manuals are available from www.compucell3d.org or at your nearest bookstore.

Potts and Lattice

- potts
- lattice_type

4.1 Potts Section

The first section of the .xml file defines the global parameters of the lattice and the simulation.

```xml
<Potts>
    <Dimensions x="101" y="101" z="1"/>
    <Anneal>0</Anneal>
    <Steps>1000</Steps>
    <FluctuationAmplitude>5</FluctuationAmplitude>
    <Flip2DimRatio>1</Flip2DimRatio>
    <Boundary_y>Periodic</Boundary_y>
    <Boundary_x>Periodic</Boundary_x>
    <NeighborOrder>2</NeighborOrder>
    <DebugOutputFrequency>20</DebugOutputFrequency>
    <RandomSeed>167473</RandomSeed>
    <EnergyFunctionCalculator Type="Statistics">
        <OutputFileName Frequency="10">statData.txt</OutputFileName>
        <OutputCoreFileNameSpinFlips Frequency="1" GatherResults="" OutputAccepted=""
→OutputRejected="" OutputTotal=""/>
    </EnergyFunctionCalculator>
</Potts>
```

This section appears at the beginning of the configuration file. Line `<Dimensions x="101" y="101" z="1"/>` declares the dimensions of the lattice to be``101 x 101 x 1``, *i.e.*, the lattice is two-dimensional and extends in the `xy` plane. The basis of the lattice is `0` in each direction, so the `101` lattice sites in the `x` and `y` directions have indices ranging from `0` to `100`. `<Steps>1000</Steps>` tells CompuCell how long the simulation lasts in MCS. After executing this number of steps, CompuCell can run simulation at zero temperature for an additional period. In our case it will run for `<Anneal>10</Anneal>` extra steps. `FluctuationAmplitude` parameter determines intrinsic fluctuation or motility of cell membrane.

Note: `FluctuationAmplitude` is a `Temperature` parameter in classical GGH model formulation. We have decided to use `FluctuationAmplitude` term instead of temperature because using word `Temperature` to describe intrinsic motility of cell membrane was quite confusing.

In the above example, fluctuation amplitude applies to all cells in the simulation. To define fluctuation amplitude separately for each cell type we use the following syntax:

```
<FluctuationAmplitude>
    <FluctuationAmplitudeParameters CellType="Condensing" FluctuationAmplitude="10"/>
    <FluctuationAmplitudeParameters CellType="NonCondensing" FluctuationAmplitude="5"/
↪>
</FluctuationAmplitude>
```

When CompuCell3D encounters expanded definition of `FluctuationAmplitude` it will use it in place of a global definition –

```
<FluctuationAmplitude>5</FluctuationAmplitude>
```

To complete the picture CompuCell3D allows users to set fluctuation amplitude individually for each cell. Using Python scripting we write:

```
for cell in self.cellList:
    if cell.type==1:
        cell.fluctAmpl=20
```

When determining which value of fluctuation amplitude to use, CompuCell first checks if `fluctAmpl` is non-negative. If this is the case it will use this value as fluctuation amplitude. Otherwise it will check if users defined fluctuation amplitude for cell types using expanded CC3DML definition and if so it will use those values as fluctuation amplitudes. Lastly, it will resort to globally defined fluctuation amplitude (`Temperature`). Thus, it is perfectly fine to use `FluctuationAmplitude` CC3DML tags and set `fluctAmpl` for certain cells. In such a case CompuCell3D will use `fluctAmpl` for cells for which users defined it and for all other cells it will use values defined in the CC3DML.

In GGH model, the fluctuation amplitude is determined taking into account fluctuation amplitude of *"source"* (expanding) cell and *"destination"* (being overwritten) cell. Currently CompuCell3D supports 3 type functions used to calculate resultant fluctuation amplitude (those functions take as argument fluctuation amplitude of "source" and *"destination"* cells and return fluctuation amplitude that is used in calculation of pixel-copy acceptance). The 3 functions are `Min`, `Max`, and `ArithmeticAverage` and we can set them using the following option of the Potts section:

```
<Potts>
    <FluctuationAmplitudeFunctionName>Min</FluctuationAmplitudeFunctionName>
    ...
</Potts>
```

By default we use `Min` function. Notice, that if you use global fluctuation amplitude definition `Temperature` it does not really matter which function you use. The differences arise when *"source"* and *"destination"* cells have different fluctuation amplitudes.

The above concepts are best illustrated by the following example:

```
<Potts>
  <Dimensions x="100" y="100" z="1"/>
  <Steps>10000</Steps>
  <FluctuationAmplitude>5</FluctuationAmplitude>
  <FluctuationAmplitudeFunctionName>ArithmeticAverage</
↪FluctuationAmplitudeFunctionName>
```

(continues on next page)

```
    <NeighborOrder>2</NeighborOrder>
</Potts>
```

Where in the CC3DML section we define global fluctuation amplitude and we also use `ArithmeticAverage` function to determine resultant fluctuation amplitude for the pixel copy.

In python script we will periodically set higher fluctuation amplitude for lattice quadrants so that when running the simulation we can see that cells belonging to different lattice quadrants have different membrane fluctuations:

```python
class FluctuationAmplitude(SteppableBasePy):
    def __init__(self, _simulator, _frequency=1):
        SteppableBasePy.__init__(self, _simulator, _frequency)

        self.quarters = [[0, 0, 50, 50], [0, 50, 50, 100], [50, 50, 100, 100], [50, 0,
→ 100, 50]]

        self.steppableCallCounter = 0

    def step(self, mcs):

        quarterIndex = self.steppableCallCounter % 4
        quarter = self.quarters[quarterIndex]

        for cell in self.cellList:

            if cell.xCOM >= quarter[0] and cell.yCOM >= quarter[1] and cell.xCOM <
→quarter[2] and cell.yCOM < quarter[3]:
                cell.fluctAmpl = 50
            else:
                # this means CompuCell3D will use globally defined
→FluctuationAmplitude
                cell.fluctAmpl = -1

        self.steppableCallCounter += 1
```

Assigning negative `fluctuationAmplitude`, `cell.fluctAmpl = -1` is interpreted by CompuCell3D as a hint to use fluctuation amplitude defined in the CC3DML.

Let us revisit our original example of the `Potts` section CC3DML:

```xml
<Potts>
    <Dimensions x="101" y="101" z="1"/>
    <Anneal>0</Anneal>
    <Steps>1000</Steps>
    <FluctuationAmplitude>5</FluctuationAmplitude>
    <Flip2DimRatio>1</Flip2DimRatio>
    <Boundary_y>Periodic</Boundary_y>
    <Boundary_x>Periodic</Boundary_x>
    <NeighborOrder>2</NeighborOrder>
    <DebugOutputFrequency>20</DebugOutputFrequency>
    <RandomSeed>167473</RandomSeed>
    <EnergyFunctionCalculator Type="Statistics">
        <OutputFileName Frequency="10">statData.txt</OutputFileName>
        <OutputCoreFileNameSpinFlips Frequency="1" GatherResults="" OutputAccepted=""
→OutputRejected="" OutputTotal=""/>
    </EnergyFunctionCalculator>
</Potts>
```

Based on discussion about the difference between pixel-flip attempts and MCS (see "Introduction to CompuCell3D") we can specify how many pixel copies should be attempted in every MCS. We specify this number indirectly by specifying the `Flip2DimRatio` by using

```
<Flip2DimRatio>1</Flip2DimRatio>
```

which tells CompuCell that it should make `1 times number of lattice sites` attempts per MCS – in our case one MCS is 101x101x1 pixel-copy attempts. To set `2.5 x 101 x 101 x 1` pixel-copy attempts per MCS you would write:

```
<Flip2DimRatio>2.5</Flip2DimRatio>
```

The line beingning with `<NeighborOrder>2</NeighborOrder>` specifies the neighbor order. The higher neighbor order the longer the Euclidian distance from a given pixel. In previous The pixel neighbors are ranked according to their distance from a reference pixel (*i.e.* the one you are measuring a distance from). thus we have 1st 2nd, 3rd and so on nearest neighbors for every pixel in the lattice. Using 1st nearest neighbor interactions may cause artifacts due to lattice anisotropy. The longer the interaction range (*i.e.* 2nd, 3rd or higher `NeighborOrder`), the more isotropic the simulation and the slower it runs. In addition, if the interaction range is comparable to the cell size, you may generate unexpected effects, since non-adjacent cells will contact each other.

On hex lattice those problems seem to be less severe and there 1st or 2nd nearest neighbor usually are sufficient.

The Potts section also contains tags called `<Boundary_y>` and `<Boundary_x>`. These tags impose boundary conditions on the lattice. In this case the x and y axes are **periodic**.

For example:

```
<Boundary_x>Periodic</Boundary_x>
```

will cause that the pixels with coordinates `x=0 , y=1, z=1` will neighbor the pixel with coordinates `x=100, y=1, z=1`. If you do not specify boundary conditions CompuCell will assume them to be of type **no-flux**, *i.e.* lattice will not be extended. The conditions are independent in each direction, so you can specify any combination of boundary conditions you like.

`DebugOutputFrequency` is used to tell CompuCell3D how often it should output text information about the status of the simulation. This tag is optional.

`RandomSeed` is used to initialize random number generator. If you do not do this all simulations will use same sequence of random numbers. Something you may want to avoid in the real simulations but is very useful while debugging your models.

`EnergyFunctionCalculator` is another option of Potts object that allows users to output statistical data from the simulation for further analysis.

Note: CC3D has the option to run in the parallel mode but output from energy calculator will only work when running in a single CPU mode.

The `OutputFileName` tag is used to specify the name of the file to which CompuCell3D will write average changes in energies returned by each plugins with corresponding standard deviations for those MCS whose values are divisible by the `Frequency` argument. Here it will write these data every 10 MCS.

A second line with `OutputCoreFileNameSpinFlips` tag is used to tell CompuCell3D to output energy change for every plugin, every pixel-copy for MCS' divisible by the frequency. Option `GatherResults=""` will ensure that there is only one file written for accepted (`OutputAccepted`), rejected (`OutputRejected`)and accepted and rejected (`OutputTotal`) pixel copies. If you will not specify `GatherResults` CompuCell3D will output separate files for different MCS's and depending on the Frequency you may end up with many files in your directory.

One option of the Potts section that we have not used here is the ability to customize acceptance function for Metropolis algorithm:

```
<Offset>-0.1</Offset>
<KBoltzman>1.2</KBoltzman>
```

This ensures that pixel copies attempts that increase the energy of the system are accepted with probability

$$P = e^{(-\Delta E - \delta)/kT} \tag{4.1}$$

where and k are specified by `Offset` and `KBoltzman` tags respectively. By default $= 0$ and $k = 1$.

As an alternative to exponential acceptance function you may use a simplified version which is essentially 1 order expansion of the exponential:

$$P = 1 - \frac{E - \delta}{kT} \tag{4.2}$$

To be able to use this function all you need to do is to add the following line in the Pots section:

```
<AcceptanceFunctionName>FirstOrderExpansion</AcceptanceFunctionName>
```

4.2 Lattice Type

Early versions of CompuCell3D allowed users to use only square lattice. Most recent versions allow the simulation to be run on hexagonal lattice as well.

Full description of hexagonal lattice including detailed derivations can be found in "Introduction to Hexagonal Lattices" available from http://www.compucell3d.org/BinDoc/cc3d_binaries/Manuals/HexagonalLattice.pdf

To enable hexagonal lattice you need to put

```
<LatticeType>Hexagonal</LatticeType>
```

in the Potts section of the CC3DML configuration file.

There are few things to be aware of when using hexagonal lattice. In 2D your pixels are hexagons but in 3D the voxels are rhombic dodecahedrons. It is particularly important to realize that surface or perimeter of the pixel (depending whether in 2D or 3D) is different than in the case of square pixel. The way CompuCell3D hex lattice implementation was done was that the volume of the pixel was constrained to be 1 regardless of the lattice type. There is also one to one correspondence between pixels of the square lattice and pixels of the hex lattice. Consequently, we can come up with transformation equations which give positions of hex pixels as a function of square lattice pixel position:

$$
\begin{cases}
[x_{hex}, y_{hex}, z_{hex}] = \left[\left(x_{cart} + \frac{1}{2}\right)L, \frac{\sqrt{3}}{2}y_{cart}L, \frac{\sqrt{6}}{3}z_{cart}L\right] \text{ for } y \mod 2 = 0 \text{ and } z \mod 3 = 0 \\
[x_{hex}, y_{hex}, z_{hex}] = \left[x_{cart}L, \frac{\sqrt{3}}{2}y_{cart}L, \frac{\sqrt{6}}{3}z_{cart}L\right] \text{ for } y \mod 2 = 1 \text{ and } z \mod 3 = 0 \\
[x_{hex}, y_{hex}, z_{hex}] = \left[x_{cart}L, \left(\frac{\sqrt{3}}{2}y_{cart} + \frac{\sqrt{3}}{6}\right)L, \frac{\sqrt{6}}{3}z_{cart}L\right] \text{ for } y \mod 2 = 0 \text{ and } z \mod 3 = 1 \\
[x_{hex}, y_{hex}, z_{hex}] = \left[\left(x_{cart} + \frac{1}{2}\right)L, \left(\frac{\sqrt{3}}{2}y_{cart} + \frac{\sqrt{3}}{6}\right)L, \frac{\sqrt{6}}{3}z_{cart}L\right] \text{ for } y \mod 2 = 1 \text{ and } z \mod 3 = 1 \\
[x_{hex}, y_{hex}, z_{hex}] = \left[x_{cart}L, \left(\frac{\sqrt{3}}{2}y_{cart} - \frac{\sqrt{3}}{6}\right)L, \frac{\sqrt{6}}{3}z_{cart}L\right] \text{ for } y \mod 2 = 0 \text{ and } z \mod 3 = 2 \\
[x_{hex}, y_{hex}, z_{hex}] = \left[\left(x_{cart} + \frac{1}{2}\right)L, \left(\frac{\sqrt{3}}{2}y_{cart} - \frac{\sqrt{3}}{6}\right)L, \frac{\sqrt{6}}{3}z_{cart}L\right] \text{ for } y \mod 2 = 1 \text{ and } z \mod 3 = 2
\end{cases}
$$

Based on the above facts one can work out how unit length and unit surface transform to the hex lattice. The conversion factors are given below:

$$S_{hex-unit} = \sqrt{\frac{2}{3\sqrt{3}}} \approx 0.6204 \tag{4.3}$$

For the 2D case, assuming that each pixel has unit volume, we get:

$$L_{hex-unit} = \sqrt{\frac{2}{\sqrt{3}}} \approx 1.075 \tag{4.4}$$

where $S_{hex-unit}$ denotes length of the hexagon and $L_{hex-unit}$ denotes a distance between centers of the hexagons. Notice that unit surface in 2D is simply a length of the hexagon side and surface area of the hexagon with side a is:

$$S = 6\frac{\sqrt{3}}{4}a^2 \tag{4.5}$$

In 3D we can derive the corresponding unit quantities starting with the formulae for volume and surface of rhombic dodecahedron (12 hedra)

$$V = \frac{16}{9}\sqrt{3}a^3$$
$$S = 8\sqrt{2}a^2$$

where a denotes length of dodecahedron edge.

Constraining the volume to be 1 we get:

$$a = \sqrt[3]{\frac{9V}{16\sqrt{3}}} \tag{4.6}$$

and thus unit surface is given by:

$$S_{unit-hex} = \frac{S}{12} = \frac{8\sqrt{2}}{12}\sqrt[3]{\frac{9V}{16\sqrt{3}}} \approx 0.445 \tag{4.7}$$

and unit length by:

$$L_{unit-hex} = 2\frac{\sqrt{2}}{\sqrt{3}}a = 2\frac{\sqrt{2}}{\sqrt{3}}\sqrt[3]{\frac{9V}{16\sqrt{3}}} \approx 1.122 \tag{4.8}$$

Plugins Section

In this section we overview CC3DML syntax for all the plugins available in CompuCell3D. Plugins are either energy functions, lattice monitors or store user assigned data that CompuCell3D uses internally to configure simulation before it is run.

- cell_type_plugin
- global_volume_and_surface_plugins
- volume_and_surface_tracker_plugins
- volume_and_surface_flex_plugins
- neighbor_tracker
- chemotaxis_plugin
- external_potential_plugin
- center_of_mass_plugin
- contact_plugin
- adhesion_flex_plugin
- compartments
- length_constraint
- connectivity
- secretion
- pde_solver_caller
- focal_point_plasticity
- curvature
- pixel_tracking_plugins
- moment_of_inertia

• convergent_extension

5.1 CellType Plugin

An example of the plugin that stores user assigned data that is used to configure simulation before it is run is a `CellType Plugin`. This plugin is responsible for defining cell types and storing cell type information. It is a basic plugin used by virtually every CompuCell simulation. The syntax is straight forward as can be seen in the example below:

```
<Plugin Name="CellType">
  <CellType TypeName="Medium" TypeId="0"/>
  <CellType TypeName="Fluid" TypeId="1"/>
  <CellType TypeName="Wall" TypeId="2" Freeze=""/>
</Plugin>
```

Here we have defined three cell types that will be present in the simulation: `Medium`, `Fluid`, `Wall`. Notice that we assign a number – `TypeId` – to every cell type. It is strongly recommended that `TypeId`'s are consecutive positive integers (e.g. `0,1,2,3...`). `Medium` is traditionally given `TypeId=0` and we recommend that you keep this convention.

Note: Important: Every CC3D simulation must define `CellType Plugin` and include at least `Medium` specification.

Notice that in the example above cell type `Wall` has extra attribute `Freeze=""`. This attribute tells CompuCell that cells of **frozen** type will not be altered by pixel copies. Freezing certain cell types is a very useful technique in constructing different geometries for simulations or for restricting ways in which cells can move.

5.2 Global Volume and Surface Constraints

One of the most commonly used energy term in the GGH Hamiltonian is a term that restricts variation of single cell volume. Its simplest form can be coded as show below:

```
<Plugin Name="Volume">
    <TargetVolume>25</TargetVolume>
    <LambdaVolume>2.0</LambdaVolume>
</Plugin>
```

By analogy we may define a term which will put similar constraint regarding the surface of the cell:

```
<Plugin Name="Surface">
    <TargetSurface>20</TargetSurface>
    <LambdaSurface>1.5</LambdaSurface>
</Plugin>
```

These two plugins inform CompuCell that the Hamiltonian will have two additional terms associated with volume and surface conservation. That is when pixel copy is attempted one cell will increase its volume and another cell will decrease. Thus overall energy of the system changes. Volume constraint essentially ensures that cells maintain the volume which close (this depends on thermal fluctuations) to target volume. The role of surface plugin is analogous to volume, that is to "preserve" surface. Note that surface plugin is commented out in the example above.

The examples shown below apply volume and surface constraints to all cells in the simulation. you can however apply volume and surface constraints to individual cell types or individual cells

---

Energy terms for volume and surface constraints have the form:

$$to$$

$$E_{volume} = \lambda_{volume} \left(V_{cell} - V_{target}\right)^2 \quad (5.1)$$

$$to$$

$$E_{surface} = \lambda_{surface} \left(S_{cell} - S_{target}\right)^2 \quad (5.2)$$

Note: Copying a single pixel may cause surface change in more that two cells – this is especially true in 3D.

5.3 VolumeTracker and SurfaceTracker plugins

These two plugins monitor lattice and update volume and surface of the cells once pixel copy occurs. In most cases users will not call those plugins directly. They will be called automatically when either Volume (calls `VolumeTracker`) or Surface (calls `SurfaceTracker`) or `CenterOfMass` (calls `VolumeTracker`) plugins are requested. However one should be aware that in some situations, for example when doing foam coarsening, where neither `Volume` nor `Surface` plugins are called, one may still want to track changes in surface or volume of cells. In such situations we explicitly invoke `VolumeTracker` or `Surface Tracker` plugin with the following syntax:

```
<Plugin Name="VolumeTracker"/>
```

or

```
<Plugin Name="SurfaceTracker"/>
```

5.4 VolumeFlex Plugin

`VolumeFlex` plugin is more sophisticated version of `Volume` Plugin. While `Volume` Plugin treats all cell types the same i.e. they all have the same target volume and lambda coefficient, `VolumeFlex` plugin allows you to assign different lambda and different target volume to different cell types. The syntax for this plugin is straightforward and essentially mimics the example below.

```
<Plugin Name="Volume">
    <VolumeEnergyParameters CellType="Prestalk" TargetVolume="68" LambdaVolume="15"/>
    <VolumeEnergyParameters CellType="Prespore" TargetVolume="69" LambdaVolume="12"/>
    <VolumeEnergyParameters CellType="Autocycling" TargetVolume="80" LambdaVolume="10
↪"/>
    <VolumeEnergyParameters CellType="Ground" TargetVolume="0" LambdaVolume="0"/>
    <VolumeEnergyParameters CellType="Wall" TargetVolume="0" LambdaVolume="0"/>
</Plugin>
```

Note: Almost all CompuCell3D modules which have options `Flex` or `LocalFlex` are implemented as a single C++ module and CC3D, based on CC3DML syntax used, figures out which functionality to load at the run time. As a result for the reminder of this reference manual we will stick to the convention that all `Flex` and `LocalFlex` modules will be invoked using core name of the module only.

Notice that in the example above cell types `Wall` and `Ground` have target volume and coefficient lambda set to 0 – very unusual. That's because in this particular case those cells are frozen so the parameters specified for these cells do not matter. In fact it is safe to remove specifications for these cell types, but just for the illustration purposes we left them here.

Using `VolumeFlex` Plugin you can effectively freeze certain cell types. All you need to do is to put very high lambda coefficient for the cell type you wish to freeze. You have to be careful though , because if initial volume of the cell of a given type is different from target volume for this cell type the cells will either shrink or expand to match target volume and only after this initial volume adjustment will they remain frozen provided `LambdaVolume` is high enough. Since rapid changes in the cell volume are uncontrolled (e.g. they can destroy many neighboring cells) you should opt for more gradual changes. In any case, we do not recommend this way of freezing cells because it is difficult to use, and also not efficient in terms of speed of simulation run.

5.5 SurfaceFlex Plugin

`SurfaceFlex` plugin is more sophisticated version of `Surface` Plugin. Everything that was said with respect to `VolumeFlex` plugin applies to `SurfaceFlex`. For syntax see example below:

```
<Plugin Name="Surface">
    <SurfaceEnergyParameters CellType="Prestalk" TargetSurface="90" LambdaSurface="0.
↪15"/>
    <SurfaceEnergyParameters CellType="Prespore" TargetSurface="98" LambdaSurface="0.
↪15"/>
    <SurfaceEnergyParameters CellType="Autocycling" TargetSurface="92" LambdaSurface=
↪"0.1"/>
    <SurfaceEnergyParameters CellType="Ground" TargetSurface="0" LambdaSurface="0"/>
    <SurfaceEnergyParameters CellType="Wall" TargetSurface="0" LambdaSurface="0"/>
</Plugin>
```

5.6 VolumeLocalFlex Plugin

`VolumeLocalFlex` Plugin is very similar to `Volume` plugin. however this time you specify lambda coefficient and target volume, individually for each cell. In the course of simulation you can change this target volume depending on e.g. concentration of *e.g.* ``FGF``in the particular cell. This way you can specify which cells grow faster, which slower based on a state of the simulation. This plugin requires you to develop a module (plugin or steppable) which will alter target volume for each cell. You can do it either in C++ or even better in Python.

Example syntax:

```
<Plugin Name="Volume"/>
```

5.7 SurfaceLocalFlex Plugin

This plugin is analogous to `VolumeLocalFlex` but operates on cell surface.

Example syntax:

```
<Plugin Name="Surface"/>
```

5.8 NeighborTracker Plugin

This plugin, as its name suggests, tracks neighbors of every cell. In addition it calculates common contact area between cell and its neighbors. We consider a neighbor this cell that has at least one common pixel side with a given cell. This means that cells that touch each other either "by edge" or by "corner" **are not** considered neighbors. See the drawing below:

5	5	5	4	4
5	5	5	4	4
5	5	4	4	4
1	1	2	2	2
1	1	2	2	2

Figure 1. Cells 5,4,1 are considered neighbors as they have non-zero common surface area. Same applies to pair of cells 4 ,2 and to 1 and 2. However, cells 2 and 5 are not neighbors because they touch each other "by corner". Notice that cell 5 has 8 pixels cell 4 , 7 pixels, cell 1 4 pixels and cell 2 6 pixels.

To include this plugin in your simulation , add the following code to the CC3DML

```
<Plugin Name="NeighborTracker"/>
```

This plugin is used as a helper module by other plugins and steppables e.g. `FocalPointPlasticity` plugin uses NeighborTracker plugin.

5.9 Chemotaxis

`Chemotaxis` plugin, is used to simulate chemotaxis of cells. For every pixel copy, this plugin calculates change of energy associated with pixel move. There are several methods to define a change in energy due to chemotaxis. By default we define a chemotaxis using the following formula:

$$\Delta E_{chem} = -\lambda \left(c(x_{destination}) - c(x_{source}) \right) \tag{5.3}$$

where $c(x_{source})$ and $c(x_{destination})$ denote chemical concentration at the pixel-copy-source and pixel-copy-destination pixel, respectively.

We also support a slight modification of the above formula in the `Chemotaxis` plugin where ΔE is non-zero only if the cell located at x_{source} *after* the pixel copy is non-medium. to enable such mode users need to include

```
<Algorithm="Regular"/>
```

tag in the body of CC3DML plugin.

Let's look at the syntax by studying the example usage of the Chemotaxis plugin:

```
<Plugin Name="Chemotaxis">
   <ChemicalField Source="FlexibleDiffusionSolverFE" Name="FGF">
      <ChemotaxisByType Type="Amoeba" Lambda="300"/>
      <ChemotaxisByType Type="Bacteria" Lambda="200"/>
   </ChemicalField>
</Plugin>
```

The body of the `Chemotaxis` plugin description contains sections called `ChemicalField`. In this section we tell CompuCell3D which module contains chemical field that we wish to use for chemotaxis. In our case it is `FlexibleDiffusionSolverFE`. Next we specify the name of the field - `FGF`. Subsequently, we specify `lambda` for each cell type so that cells of different type may respond differently to a given chemical. In particular types not listed will not respond to chemotaxis at all.

Occasionally we may want to use different formula for the chemotaxis than the one presented above. Current version of CompCell3D supports the following definitions of change in chemotaxis energy (`Saturation` and `SaturationLinear` respectively):

$$\Delta E_{chem} = -\lambda \left[\frac{c(x_{destination})}{s + c(x_{destination})} - \frac{c(x_{source})}{s + c(x_{source})} \right] \tag{5.4}$$

or

$$\Delta E_{chem} = -\lambda \left[\frac{c(x_{destination})}{sc(x_{destination}) + 1} - \frac{c(x_{source})}{sc(x_{source}) + 1} \right] \tag{5.5}$$

where `s` denotes saturation constant. To use first of the above formulas we set the value of the saturation coefficient:

```
<Plugin Name="Chemotaxis">
   <ChemicalField Source="FlexibleDiffusionSolverFE" Name="FGF">
      <ChemotaxisByType Type="Amoeba" Lambda="0"/>
      <ChemotaxisByType Type="Bacteria" Lambda="2000000" SaturationCoef="1"/>
   </ChemicalField>
</Plugin>
```

Notice that this only requires small change in line where you previously specified only lambda.

```
<ChemotaxisByType Type="Bacteria" Lambda="2000000" SaturationCoef="1"/>
```

To use second of the above formulas use `SaturationLinearCoef` instead of `SaturationCoef`:

```
<Plugin Name="Chemotaxis">
   <ChemicalField Source="FlexibleDiffusionSolverFE" Name="FGF">
      <ChemotaxisByType Type="Amoeba" Lambda="0"/>
      <ChemotaxisByType Type="Bacteria" Lambda="2000000" SaturationLinearCoef="1"/>
   </ChemicalField>
</Plugin>
```

Sometimes it is desirable to have chemotaxis **at the interface between** only certain types of cells **and not between** other cell-type-pairs. In such a case we augment `ChemotaxisByType` element with the following attribute:

```
<ChemotaxisByType Type="Amoeba" Lambda="100" "ChemotactTowards="Medium"/>
```

This will cause that the change in chemotaxis energy will be non-zero only for those pixel copy attempts that happen between pixels belonging to `Amoeba` and `Medium`.

Note: The term `ChemotactTowards` means "chemotax at the interface between"

The definitions of chemotaxis presented so far do not allow specification of chemotaxis parameters individually for each cell. To do this we will use Python scripting. We still need to specify in the CC3DML which fields are important from chamotaxis stand point. Only fields listed in the CC3DML will be used to calculate chemotaxis energy:

```
...

<Plugin Name="CellType">
    <CellType TypeName="Medium" TypeId="0"/>
    <CellType TypeName="Bacterium" TypeId="1" />
    <CellType TypeName="Macrophage" TypeId="2"/>
    <CellType TypeName="Wall" TypeId="3" Freeze=""/>
</Plugin>

...

<Plugin Name="Chemotaxis">
    <ChemicalField Source="FlexibleDiffusionSolverFE" Name="ATTR">
    <ChemotaxisByType Type="Macrophage" Lambda="20"/>
    </ChemicalField>
</Plugin>

...
```

In the above excerpt from the CC3DML configuration file we see that cells of type `Macrophage` will chemotax in response to `ATTR` gradient.

Using Python scripting we can modify chemotaxis properties of individual cells as follows:

```python
class ChemotaxisSteering(SteppableBasePy):
        def __init__(self, _simulator, _frequency=100):
            SteppableBasePy.__init__(self, _simulator, _frequency)

        def start(self):

            for cell in self.cellList:
                if cell.type == 2:
                    cd = self.chemotaxisPlugin.addChemotaxisData(cell, "ATTR")
                    cd.setLambda(20.0)

                    # cd.initializeChemotactTowardsVectorTypes("Bacterium,Medium")
                    cd.assignChemotactTowardsVectorTypes([0, 1])

                    break

        def step(self, mcs):
            for cell in self.cellList:
                if cell.type == 2:

                    cd = self.chemotaxisPlugin.getChemotaxisData(cell, "ATTR")
                    if cd:
                        lam = cd.getLambda() - 3
```

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```
                cd.setLambda(lam)
            break
```

In the `start` function for first encountered cell of type `Macrophage` (`type==2`) we insert `ChemotaxisData` object (it determines chemotaxing properties) and initialize λ parameter to `20`. We also initialize vector of cell types towards which Macrophage cell will chemotax (it will chemotax towards Medium and Bacterium cells). Notice the `break` statement inside the if statement, inside the loop. It ensures that only first encountered Macrophage cell will have chemotaxing properties altered.

In the step function we decrease lambda chemotaxis by `3` units every `100` MCS. In effect we turn a cell from chemotaxing up `ATTR` gradient to being chemorepelled.

In the above example we have more than one macrophage but only one of them has altered chemotaxing properties. The other macrophages have chemotaxing properties set in the CC3DML section. CompuCell3D first checks if local definitions of chemotaxis are available (i.e. for individual cells) and if so it uses those. Otherwise it will use definitions from from the CC3DML.

The `ChemotaxisData` structure has additional functions which allow to set chemotaxis formula used. For example we may type:

```python
def start(self):
    for cell in self.cellList:
        if cell.type == 2:
            cd = self.chemotaxisPlugin.addChemotaxisData(cell, "ATTR")
            cd.setLambda(20.0)
            cd.setSaturationCoef(200.0)

            # cd.initializeChemotactTowardsVectorTypes("Bacterium,Medium")
            cd.assignChemotactTowardsVectorTypes([0, 1])

            break
```

to activate `Saturation` formula. To activate `SaturationLinear` formula we would use:

```
cd.setSaturationLinearCoef(2.0)
```

> **Warning:** When you use chemotaxis plugin you have to make sure that fields that you refer to and module that contains this fields are declared in the CC3DML file. Otherwise you will most likely cause either program crash (which is not as bad as it sounds) or unpredicted behavior (much worse scenario, although unlikely as we made sure that in the case of undefined symbols, CompuCell3D exits)

5.10 ExternalPotential Plugin

`Chemotaxis` plugin is used to cause directional cell movement in response to chemical gradient. Another way to achieve directional movement is to use `ExternalPotential` plugin. This plugin is responsible for imposing a directed pressure (or rather force) on cells. It is used to implement persistent motion of cells and its applications can be very diverse.

Example usage of this plugin looks as follows:

```xml
<Plugin Name="ExternalPotential">
   <Lambda x="-0.5" y="0.0" z="0.0"/>
</Plugin>
```

`Lambda` is a vector quantity and determines components of force along three axes. In this case we apply force along `x` pointing in the positive direction.

Note: Positive component of Lambda vector pushes cell in the negative direction and negative component pushes cell in the positive direction

---

We can also apply external potential to specific cell types:

```
<Plugin Name="ExternalPotential">
    <ExternalPotentialParameters CellType="Body1" x="-10" y="0" z="0"/>
    <ExternalPotentialParameters CellType="Body2" x="0" y="0" z="0"/>
    <ExternalPotentialParameters CellType="Body3" x="0" y="0" z="0"/>
</Plugin>
```

Where in `ExternalPotentialParameters` we specify which cell type is subject to external potential (`Lambda` is specified using `x` , `y` , `z` attributes).

We can also apply external potential to individual cells. In that case, in the CC3DML section we only need to specify:

```
<Plugin Name="ExternalPotential"/>
```

and in the Python file we change `lambdaVecX`, `lambdaVecY`, `lambdaVecZ`, which are properties of cell. For example in Python we could write:

```
cell.lambdaVecX = -10
```

Calculations done by `ExternalPotential` Plugin are by default based on direction of pixel copy (similarly as in chemotaxis plugin). One can, however, force CC3D to do calculations based on movement of center of mass of cell. To use algorithm based on center of mass movement we use the following CC3DML syntax:

```
<Plugin Name="ExternalPotential">

    <Algorithm>CenterOfMassBased</Algorithm>

    ...

</Plugin>
```

Note: In the pixel-based algorithm the typical value of pixel displacement used in calculations is of the order of `1` (pixel) whereas typical displacement of center of mass of cell due to single pixel copy is of the order of 1/cell volume (pixels) – ~ `0.1` pixel. This implies that to achieve compatible behavior of cells when using center of mass algorithm we need to multiply `lambda`'s by appropriate factor, typically of the order of `10`.

5.10. ExternalPotential Plugin

5.11 CenterOfMass Plugin

`CenterOfMass` plugin monitors changes n the lattice and updates centroids of the cell:

$$x_{CM}^{centroid} = \sum_i x_i$$

$$y_{CM}^{centroid} = \sum_i y_i$$

$$z_{CM}^{centroid} = \sum_i z_i$$

where `i` denotes pixels belonging to a given cell. To obtain coordinates of a center of mass of a given cell we divide centroids by cell volume:

$$x_{CM} = \frac{x_{CM}^{centroid}}{V}$$

$$y_{CM} = \frac{y_{CM}^{centroid}}{V}$$

$$z_{CM} = \frac{z_{CM}^{centroid}}{V}$$

This plugin is aware of boundary conditions and centroids are calculated properly regardless which boundary conditions are used. The CC3DML syntax is very simple:

```
<Plugin Name="CenterOfMass"/>
```

To access center of mass coordinates from Python we use the following syntax:

```
print('x-component of COM is:', cell.xCOM)
print ('y-component of COM is:', cell.yCOM)
print ('z-component of COM is:', cell.zCOM)
```

> **Warning:** Center of mass parameters in Python are read only. Any attempt to modify them will likely mess up the simulation.

5.12 Contact Plugin

Contact plugin implements computations of adhesion energy between neighboring cells.

Together with volume constraint contact energy is one of the most commonly used energy terms in the GGH Hamiltonian. In essence it describes how cells "stick" to each other.

The explicit formula for the energy is:

$$E_{adhesion} = \sum_{i,j,neighbors} J\left(\tau_{\sigma(i)}, \tau_{\sigma(j)}\right)\left(1 - \delta_{\sigma(i),\sigma(j)}\right) \tag{5.6}$$

where `i` and `j` label two neighboring lattice sites σ's denote cell Ids, τ's denote cell types .

In the above formula, we need to differentiate between cell types and cell Ids. This formula shows that cell types and cell Ids **are not the same**. The `Contact` plugin in the `.xml` file, defines the energy per unit area of contact between cells of different types ($J\left(\tau_{\sigma(i)}, \tau_{\sigma(j)}\right)$) and the interaction range `NeighborOrder` of the contact:

```
<Plugin Name="Contact">
    <Energy Type1="Foam" Type2="Foam">3</Energy>
    <Energy Type1="Medium" Type2="Medium">0</Energy>
    <Energy Type1="Medium" Type2="Foam">0</Energy>
    <NeighborOrder>2</NeighborOrder>
</Plugin>
```

In this case, the interaction range is 2, thus only up to second nearest neighbor pixels of a pixel undergoing a change or closer will be used to calculate contact energy change. `Foam` cells have contact energy per unit area of 3 and `Foam` and `Medium` as well as Medium and Medium have contact energy of 0 per unit area. For more information about contact energy calculations see "Introduction to CompuCell3D"

5.13 AdhesionFlex Plugin

`AdhesionFlex` offers a very flexible way to define adhesion between cells. It allows setting individual adhesivity properties for each cell. Users can use either CC3DML syntax or Python scripting to initialize adhesion molecule density for each cell. In addition, `Medium` can also carry its own adhesion molecules. We use the following formula to calculate adhesion energy in `AdhesionFlex` plugin:

$$E_{adhesion} = \sum_{i,j,neighbors} \left(-\sum_{m,n} k_{mn} F\left(N_m\left(i\right), N_n\left(i\right)\right) \right) \left(1 - \delta_{\sigma(i),\sigma(j)}\right) \tag{5.7}$$

where indexes `i`, `j` label pixels, $-\sum_{m,n} k_{mn} F\left(N_m\left(i\right), N_n\left(i\right)\right)$ denotes contact energy between cell types $\sigma(i)$ and $\sigma(j)$ and indexes `m`, `n` label adhesion molecules in cells composed of pixels `i` and `j` respectively. `F` denotes user-defined function of N_m and N_n. Although this may look a bit complex, the basic idea is simple: each cell has certain number of adhesion molecules on its surface. When cells touch each other the resultant energy is simply a "product of interactions" of adhesion molecules from one cell with adhesion molecules from another cell. The CC3DML syntax for this plugin is given below:

```
<Plugin Name="AdhesionFlex">
    <AdhesionMolecule Molecule="NCad"/>
    <AdhesionMolecule Molecule="NCam"/>
    <AdhesionMolecule Molecule="Int"/>
    <AdhesionMoleculeDensity CellType="Cell1" Molecule="NCad" Density="6.1"/>
    <AdhesionMoleculeDensity CellType="Cell1" Molecule="NCam" Density="4.1"/>
    <AdhesionMoleculeDensity CellType="Cell1" Molecule="Int" Density="8.1"/>
    <AdhesionMoleculeDensity CellType="Medium" Molecule="Int" Density="3.1"/>
    <AdhesionMoleculeDensity CellType="Cell2" Molecule="NCad" Density="2.1"/>
    <AdhesionMoleculeDensity CellType="Cell2" Molecule="NCam" Density="3.1"/>

    <BindingFormula Name="Binary">
        <Formula> min(Molecule1,Molecule2)</Formula>
        <Variables>
            <AdhesionInteractionMatrix>
             <BindingParameter Molecule1="NCad" Molecule2="NCad">-1.0</
→BindingParameter>
                <BindingParameter Molecule1="NCam" Molecule2="NCam">2.0</
→BindingParameter>
                <BindingParameter Molecule1="NCad" Molecule2="NCam">-10.0</
→BindingParameter>
                <BindingParameter Molecule1="Int" Molecule2="Int">-10.0</
→BindingParameter>
            </AdhesionInteractionMatrix>
        </Variables>
```

```
    </BindingFormula>

    <NeighborOrder>2</NeighborOrder>
</Plugin>
```

k:sub:'mn' matrix is specified within the `AdhesionInteractionMatrix` tag – the elements are listed using `BindingParameter` tags. The `AdhesionMoleculeDensity` tag specifies initial concentration of adhesion molecules. Even if you are going to modify those from Python you are still required to specify the names of adhesion molecules and associate them with appropriate cell types. Failure to do so may result in simulation crash or undefined behaviors. The user-defined function *F* is specified using `Formula` tag where the arguments of the function are called `Molecule1` and `Molecule2`. The syntax has to follow syntax of the muParser - http://muparser.sourceforge.net/mup_features.html#idDef2 .

CompuCell3D example – *Demos/AdhesionFlex* - demonstrates how to manipulate concentration of adhesion molecules. For example:

```
self.adhesionFlexPlugin.getAdhesionMoleculeDensity(cell,"NCad")
```

allows to access adhesion molecule concentration using its name (as given in the CC3DML above using `AdhesionMoleculeDensity` tag).

```
self.adhesionFlexPlugin.getAdhesionMoleculeDensityByIndex(cell,1)
```

allows to access adhesion molecule concentration using its index in the adhesion molecule density vector. The order of the adhesion molecule densities in the vector is the same as the order in which they were declared in the CC3DML above - AdhesionMoleculeDensity tags.

```
self.adhesionFlexPlugin.getAdhesionMoleculeDensityVector(cell)
```

allows access to entire adhesion molecule density vector. Each of these functions has its corresponding function which operates on `Medium`. In this case we do not give cell as first argument:

```
self.adhesionFlexPlugin.getMediumAdhesionMoleculeDensity('Int')

self.adhesionFlexPlugin.getMediumAdhesionMoleculeDensityByIndex (0)

self.adhesionFlexPlugin.getMediumAdhesionMoleculeDensityVector(cell)
```

To change the value of the adhesion molecule density we use set functions:

Notice that in this last function we passed entire Python list as the argument. CC3D will check if the number of entries in this vector is the same as the number of entries in the currently used vector. If so the values from the passed vector will be copied, otherwise they will be **ignored**.

Note: During mitosis we create new cell (`childCell`) and the adhesion molecule vector of this cell will have no components. However in order for simulation to continue we have to initialize this vector with number of adhesion molecules appropriate to `childCell` type. We know that `setAdhesionMoleculeDensityVector` is not appropriate for this task so we have to use:

```
self.adhesionFlexPlugin.assignNewAdhesionMoleculeDensityVector(cell,[3.4,2.1,12.1])
```

which will ensure that the content of passed vector is copied entirely into cell's vector (making size adjustments as necessary).

Note: You have to make sure that the number of newly assigned adhesion molecules is exactly the same as the number of adhesion molecules declared for the cell of this particular type.

All of the `get` functions has corresponding set function which operates on `Medium`:

```
self.adhesionFlexPlugin.setMediumAdhesionMoleculeDensity("NCam",2.8)

self.adhesionFlexPlugin.setMediumAdhesionMoleculeDensityByIndex(2,16.8)

self.adhesionFlexPlugin.setMediumAdhesionMoleculeDensityVector([1.4,3.1,18.1])

self.adhesionFlexPlugin.assignNewMediumAdhesionMoleculeDensityVector([1.4,3.1,18.1])
```

5.14 Compartmentalized cells. ContactInternal Plugin

Calculating contact energies between compartmentalized cells is analogous to the non-compartmentalized case. The energy expression takes the following form:

$$E_{adhesion} = \sum_{i,j,neighbors} J\left(\sigma(\mu_i, \nu_i), \sigma(\mu_j, \nu_j)\right)$$
(5.8)

where `i` and `j` denote pixels , $\sigma(\mu,\nu)$ denotes a cell type of a cell with cluster id μ and cell id ν . In compartmental cell models a cell is a collection of subcells. Each subcell has a unique id ν (cell id). In addition to that each subcell will have additional attribute, a cluster id (μ) that determines to which cluster of subcells a given subcell belongs. Tthink of a cluster as a cell with nonhomogenous cytoskeleton. So a cluster corresponds to a biological cell and subcells (or compartments) represents parts of a cell e.g. a nucleus. The core idea here is to have different contact energies between subcells belonging to the same cluster and different energies for cells belonging to different clusters. Technically subcells of a cluster are "regular" CompuCell3D cells. By giving them an extra attribute cluster id ((μ) we can introduce a concept of compartmental cells. In our convention $\sigma(0,0)$ denotes medium
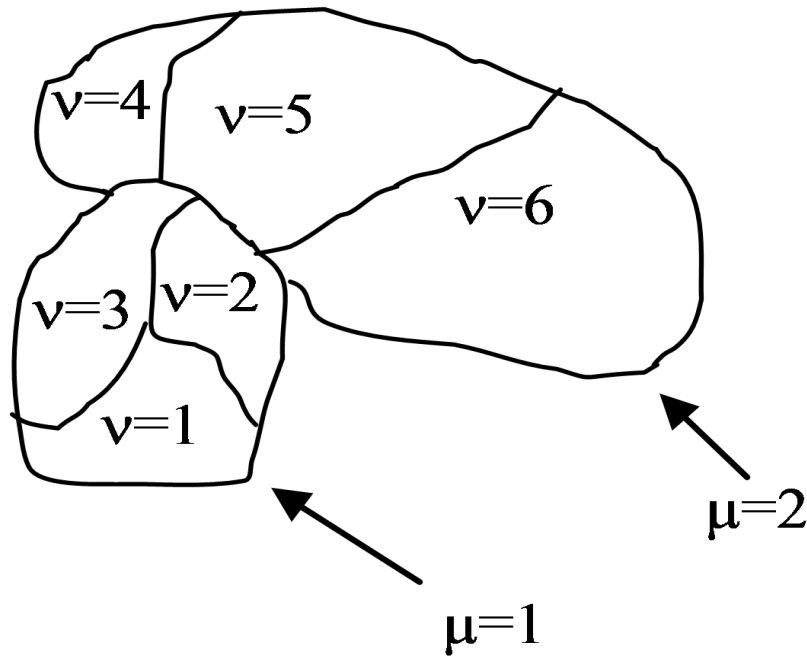
Figure 2. Two compartmentalized cells (cluster id $\mu = 1$ and cluster id $\mu = 2$). Compartmentalized cell $\mu = 1$ consists of subcells with cell id $\nu = 1, 2, 3$ and compartmentalized cell $\mu = 2$ consists of subcells with cell id $\nu = 4, 5, 6$

Introduction of cluster id and cell id are essential for the definition of $J\left(\sigma(\mu_i, \nu_i), \sigma(\mu_j, \nu_j)\right)$

Indeed: $\begin{cases} J\left(\sigma(\mu_i, \nu_i), \sigma(\mu_j, \nu_j)\right) = J^{external}\left(\sigma(\mu_i, \nu_i), \sigma(\mu_j, \nu_j)\right) & \text{if } \mu_i \neq \mu_j \\ J\left(\sigma(\mu_i, \nu_i), \sigma(\mu_j, \nu_j)\right) = J^{internal}\left(\sigma(\mu_i, \nu_i), \sigma(\mu_j, \nu_j)\right) & \text{if } \mu_i = \mu_j \end{cases}$ As we can see from above there

are two hierarchies of contact energies – external and internal. To describe adhesive interactions between different compartmentalized cells we use two plugins: `Contact` and `ContactInternal`. `Contact` plugin calculates energy between two cells belonging to different clusters and `ContactInternal` calculates energies between cells belonging to the same cluster. An example syntax is shown below

```
<Plugin Name="Contact">

    <Energy Type1="Base" Type2="Base">0</Energy>
    <Energy Type1="Top" Type2="Base">25</Energy>
    <Energy Type1="Center" Type2="Base">30</Energy>
    <Energy Type1="Bottom" Type2="Base">-2</Energy>
    <Energy Type1="Side1" Type2="Base">25</Energy>
    <Energy Type1="Side2" Type2="Base">25</Energy>
    <Energy Type1="Medium" Type2="Base">0</Energy>


    <Energy Type1="Medium" Type2="Medium">0</Energy>
    <Energy Type1="Top" Type2="Medium">30</Energy>
    <Energy Type1="Bottom" Type2="Medium">20</Energy>
    <Energy Type1="Side1" Type2="Medium">30</Energy>
    <Energy Type1="Side2" Type2="Medium">30</Energy>
```

```
    <Energy Type1="Center" Type2="Medium">45</Energy>

    <Energy Type1="Top" Type2="Top">2</Energy>
    <Energy Type1="Top" Type2="Bottom">100</Energy>
    <Energy Type1="Top" Type2="Side1">25</Energy>
    <Energy Type1="Top" Type2="Side2">25</Energy>
    <Energy Type1="Top" Type2="Center">35</Energy>

    <Energy Type1="Bottom" Type2="Bottom">10</Energy>
    <Energy Type1="Bottom" Type2="Side1">25</Energy>
    <Energy Type1="Bottom" Type2="Side2">25</Energy>
    <Energy Type1="Bottom" Type2="Center">35</Energy>

    <Energy Type1="Side1" Type2="Side1">25</Energy>
    <Energy Type1="Side1" Type2="Center">25</Energy>
    <Energy Type1="Side2" Type2="Side2">25</Energy>
    <Energy Type1="Side2" Type2="Center">25</Energy>
    <Energy Type1="Side1" Type2="Side2">15</Energy>

    <Energy Type1="Center" Type2="Center">20</Energy>

    <NeighborOrder>2</NeighborOrder>
</Plugin>
```

and

```
<Plugin Name="ContactInternal">

    <Energy Type1="Base" Type2="Base">0</Energy>
    <Energy Type1="Base" Type2="Bottom">0</Energy>
    <Energy Type1="Base" Type2="Side1">0</Energy>
    <Energy Type1="Base" Type2="Side2">0</Energy>
    <Energy Type1="Base" Type2="Center">0</Energy>

    <Energy Type1="Top" Type2="Top">4</Energy>
    <Energy Type1="Top" Type2="Bottom">25</Energy>
    <Energy Type1="Top" Type2="Side1">22</Energy>
    <Energy Type1="Top" Type2="Side2">22</Energy>
    <Energy Type1="Top" Type2="Center">15</Energy>

    <Energy Type1="Bottom" Type2="Bottom">4</Energy>
    <Energy Type1="Bottom" Type2="Side1">15</Energy>
    <Energy Type1="Bottom" Type2="Side2">15</Energy>
    <Energy Type1="Bottom" Type2="Center">10</Energy>

    <Energy Type1="Side1" Type2="Side1">11</Energy>
    <Energy Type1="Side2" Type2="Side2">11</Energy>
    <Energy Type1="Side1" Type2="Side2">11</Energy>

    <Energy Type1="Side2" Type2="Center">10</Energy>
    <Energy Type1="Side1" Type2="Center">10</Energy>

    <Energy Type1="Center" Type2="Center">2</Energy>

    <NeighborOrder>2</NeighborOrder>
</Plugin>
```

Depending whether pixels for which we calculate contact energies belong to the same cluster or not we will use internal or external contact energies respectively.

5.15 LengthConstraint Plugin

This plugin imposes elongation constraint on the cell. It "measures" a cell along its "axis of elongation" and ensures that cell length along the elongation axis is close to target length. For detailed description of this algorithm in 2D see Roeland Merks' paper "Cell elongation is a key to in silico replication of in vitro vasculogenesis and subsequent remodeling" Developmental Biology **289** (2006) 44-54). This plugin is usually used in conjunction with Connectivity Plugin or ConnectivityGlobal Plugin. The syntax is as follows:

```
<Plugin Name="LengthConstraint">
    <LengthEnergyParameters CellType="Body1" TargetLength="30" LambdaLength="5"/>
</Plugin>
```

`LambdaLength` determines the degree of cell length oscillation around `TargetLength` parameter. The higher `LambdaLength` the less freedom a cell will have to deviate from `TargetLength`.

In the 3D case we use the following syntax:

```
<Plugin Name="LengthConstraint">
    <LengthEnergyParameters CellType="Body1" TargetLength="20" MinorTargetLength="5"␣
→LambdaLength="100" />
</Plugin>
```

Notice new attribute called `MinorTargetLength`. In 3D it is not sufficient to constrain the "length" of the cell you also need to constrain "width" of the cell along axis perpendicular to the major axis of the cell. This "width" is referred to as `MinorTargetLength`.

The parameters are assigned using Python – see *Demos/elongationFlexTest* example.

To control length constraint individually for each cell we may use Python scripting to assign `LambdaLength`, `TartgetLength` and in 3D `MinorTargetLength`. In Python steppable we typically would write the following code:

```
self.lengthConstraintPlugin.setLengthConstraintData(cell,10,20)
```

which sets length constraint for `cell` by setting `LambdaLength=10` and `TargetLength=20`. In 3D we may specify`` MinorTargetLength`` (we set it to 5) by adding 4th parameter to the above call:

```
self.lengthConstraintPlugin.setLengthConstraintData(cell,10,20,5)
```

Note: If we use CC3DML specification of length constraint for certain cell types and in Python we set this constraint individually for a single cell then the local definition of the constraint has priority over definitions for the cell type.

If, in the simulation, we will be setting length constraint for only few individual cells then it is best to manipulate the constraint parameters from the Python script. In this case in the CC3DML we only have to declare that we will use length constraint plugin and we may skip the definition by-type definitions:

```
<Plugin Name="LengthConstraint"/>
```

Note: IMPORTANT: When using target length plugins it is important to use connectivity constraint. This constraint will check if a given pixel copy breaks cell connectivity. If it does, the plugin will add large energy penalty (defined

by a user) to change of energy effectively prohibiting such pixel copy. In the case of 2D on square lattice checking cell connectivity can be done locally and thus is very fast. In 3D the connectivity algorithm is a bit slower but but its performance is acceptable. Therefore if you need large cell elongations you should always use connectivity in order to prevent cell fragmentation

5.16 Connectivity Plugins

The "basic" `Connectivity` plugin works **only in 2D and only on square lattice** and is used to ensure that cells are connected or in other words to prevent separation of the cell into pieces. The detailed algorithm for this plugin is described in Roeland Merks' paper "Cell elongation is a key to *in-silico* replication of in vitro vasculogenesis and subsequent remodeling" Developmental Biology **289** (2006) 44-54). There was one modification of the algorithm as compared to the paper. Namely, to ensure proper connectivity we had to reject all pixel copies that resulted in more that two collisions. (see the paper for detailed explanation what this means).

The syntax of the plugin is straightforward:

Note: The value of `Penalty` is irrelevant because all `Connectivity` plugins have a special status. Namely, CC3D will call connectivity plugin to check if the a given pixel copy would lead to cell fragmentation and if so it will reject the pixel copy without computing energy-based pixel-copy acceptance function. thus the only thing that matters here is that penalty parameter is a positive number. Any number

As we mentioned, earlier 2D connectivity algorithm is particularly fast but works only on Cartesian lattice nad in 2D only. If you are on hex lattice or are working with 3 dimensions You should use `ConnectivityGlobal` plugin and there specify so called `FastAlgorithm` to get decent performance.

For example (see *Demos/PluginDemos/connectivity_global_fast*):

```
<Plugin Name="ConnectivityGlobal">
    <FastAlgorithm/>
    <ConnectivityOn Type="NonCondensing"/>
    <ConnectivityOn Type="Condensing"/>
</Plugin>
```

will enforce connectivity of cells of type `Condensing` and `NonCondensing` and will use "fast algorithm".

If you want to enforce connectivity for individual cells cells (see *Demos/PluginDemos/connectivity_elongation_fast*) you would use the following CC3DML code:

```
<Plugin Name="ConnectivityGlobal">
    <FastAlgorithm/>
</Plugin>
```

and couple it with the following Python steppable:

```
class ConnectivityElongationSteppable(SteppableBasePy):
    def __init__(self,_simulator,_frequency=10):
        SteppableBasePy.__init__(self,_simulator,_frequency)

    def start(self):
        for cell in self.cellList:
            if cell.type==1:
                cell.connectivityOn = True
```

(continues on next page)

```
        elif cell.type==2:
            cell.connectivityOn = True
```

Below we describe a slower version of ConnectivityGlobal plugin that is still supported but has much slower performance and for that reason we encourage you to try faster implementation described above

---

Note: DEPRECATED

A more general type of connectivity constraint is implemented in `ConnectivityGlobal` plugin. In this case we calculate volume of a cell using breadth first search algorithm and compare it with actual volume of the cell. If they agree we conclude that cell connectivity is preserved. This plugin works both in 2D and 3D and on either type of lattice. However, the computational cost of running such algorithm can be quite high so it is best to limit this plugin to cell types for which connectivity of cell is really essential:

```
<Plugin Name="ConnectivityGlobal">
    <Penalty Type="Body1">1000000000</Penalty>
</Plugin>
```

As we mentioned before the actual value of `Penalty` parameter does not matter as long it is a positive number

In certain types of simulation it may happen that at some point cells change cell types. If a cell that was not subject to connectivity constraint, changes type to the cell that is constrained by global connectivity and this cell is fragmented before type change this situation normally would result in simulation freeze. However, CompuCell3D, first before applying constraint it will check if the cell is fragmented. If it is, there is no constraint. Global connectivity constraint is only applied when cell is non-fragmented.

Quite often in the simulation we don't need to impose connectivity constraint on all cells or on all cells of given type. Usually only select cell types or select cells are elongated and therefore need connectivity constraint. In such a case we simply declare `ConnectivityGlobal` with no further specifications taking place in CC3DML The actual connectivity assignments to particular cells take place in Python

In CC3DML we only declare:

```
<Plugin Name="ConnectivityGlobal"/>
```

In Python we manipulate/access connectivity parameters for individual cells using the following syntax:

```
class ElongationFlexSteppable(SteppableBasePy):
    def __init__(self,_simulator,_frequency=10):
        SteppableBasePy.__init__(self, _simulator, _frequency)
        # self.lengthConstraintPlugin=CompuCell.getLengthConstraintPlugin()


    def start(self):
        pass


    def step(self,mcs):
        for cell in self.cellList:
            if cell.type==1:
                self.lengthConstraintPlugin.setLengthConstraintData(cell,20,
→20) # cell , lambdaLength, targetLength
                self.connectivityGlobalPlugin.setConnectivityStrength(cell,
→10000000) #cell, strength

            elif cell.type==2:
```

```
                self.lengthConstraintPlugin.setLengthConstraintData(cell,20,
→30)   # cell , lambdaLength, targetLength
                self.connectivityGlobalPlugin.setConnectivityStrength(cell,
→10000000) #cell, strength
```

See also example in *Demos/PluginDemos/elongationFlexTest.*

If you are in 2D and on Cartesian lattice you may instead use `ConnectivityLocalFlex`

In this case

In CC3DML we only declare:

```
<Plugin Name="ConnectivityLocalFlex"/>
```

and in Python:

```
self.connectivityLocalFlexPlugin.setConnectivityStrength(cell,20.7)
self.connectivityLocalFlexPlugin.getConnectivityStrength(cell)
```

5.17 Secretion / SecretionLocalFlex Plugin

Secretion "by cell type" can and should be handled by the appropriate PDE solver. To implement secretion in individual cells using Python we can use secretion plugin defined in the CC3DML as:

```
<Plugin Name="Secretion"/>
```

or as:

```
<Plugin Name="SecretionLocalFlex"/>
```

The inclusion of the above code in the CC3DML will allow users to implement secretion for individual cells from Python.

Note: Secretion for individual cells invoked via Python will be called only once per MCS.

Warning: Secretion plugin can be used to implement secretion by cell type however **we strongly advise against doing so**. Defining secretion by cell type in the `Secretion` plugin will lead to performance degradation on multi-core machines. Please see section below for more information if you are still interested in using secretion by cell-type inside `Secretion` plugin

Typical use of secretion from Python is demonstrated best in the example below:

```python
class SecretionSteppable(SecretionBasePy):
    def __init__(self, _simulator, _frequency=1):
        SecretionBasePy.__init__(self, _simulator, _frequency)

    def step(self, mcs):
        attrSecretor = self.getFieldSecretor("ATTR")
        for cell in self.cellList:
```

```python
        if cell.type == 3:
            attrSecretor.secreteInsideCell(cell, 300)
            attrSecretor.secreteInsideCellAtBoundary(cell, 300)
            attrSecretor.secreteOutsideCellAtBoundary(cell, 500)
            attrSecretor.secreteInsideCellAtCOM(cell, 300)
        elif cell.type == 2:
            attrSecretor.secreteInsideCellConstantConcentration(cell, 300)
```

Note: Instead of using `SteppableBasePy` class we are using `SecretionBasePy` class. The reason for this is that in order for secretion plugin with secretion modes accessible from Python to behave exactly as previous versions of PDE solvers (where secretion was done first followed by the "diffusion" step) we have to ensure that secretion steppable implemented in Python is called **before** each Monte Carlo Step, which implies that it will be also called before "diffusing" function of the PDE solvers. `SecretionBasePy` sets extra flag which ensures that steppable which inherits from `SecretionBasePy` is called before MCS (and before all "regular" Python steppables).

There is no magic to `SecretionBasePy` - if you still want to use `SteppableBasePy` as a base class for secretion do so, but remember that you need to set flag:

```python
self.runBeforeMCS=1
```

to ensure that your new steppable will run before each MCS. See example below for alternative implementation of `SecretionSteppable` using `SteppableBasePy` as a base class:

```python
class SecretionSteppable(SteppableBasePy):
    def __init__(self,_simulator,_frequency=1):
        SteppableBasePy.__init__(self,_simulator, _frequency)
        self.runBeforeMCS=1
    def step(self,mcs):
        attrSecretor=self.getFieldSecretor("ATTR")
        for cell in self.cellList:
            if cell.type==3:
                attrSecretor.secreteInsideCell(cell,300)
                attrSecretor.secreteInsideCellAtBoundary(cell,300)
                attrSecretor.secreteOutsideCellAtBoundary(cell,500)
                attrSecretor.secreteOutsideCellAtBoundaryOnContactwith(cell,500,[2,3])
                attrSecretor.secreteInsideCellAtCOM(cell,300)
                attrSecretor.uptakeInsideCellAtCOM(cell,300,0.2)
            elif cell.type==2:
                attrSecretor.secreteInsideCellConstantConcentration(cell,300)
```

The secretion of individual cells is handled through `FieldSecretor` objects. `FieldSecretor` concept is quite convenient because the amount of Python coding is quite small. To secrete chemical (this is now done for individual cell) we first create field secretor object:

```python
attrSecretor = self.getFieldSecretor("ATTR")
```

which allows us to secrete into field called `ATTR`.

Then we pick a cell and using field secretor we simulate secretion of chemical `ATTR` by a cell:

```python
attrSecretor.secreteInsideCell(cell,300)
```

Currently we support 7 secretion modes for individual cells:

1. `secreteInsideCell` – this is equivalent to secretion in every pixel belonging to a cell

2. `secreteInsideCellConstantConcentration` – this is equivalent to secretion in every pixel belonging to a cell and setting concentration to fixed, constant level

3. `secreteInsideCellAtBoundary` – secretion takes place in the pixels belonging to the cell boundary

4. `secreteInsideCellAtBoundaryOnContactWith` - secretion takes place in the pixels belonging to the cell boundary that touches any of the cells listed as the last argument of the function call

5. `secreteOutsideCellAtBoundary` – secretion takes place in pixels which are outside the cell but in contact with cell boundary pixels

6. `secreteOutsideCellAtBoundaryOnContactWith` - secretion takes place in pixels which are outside the cell but in contact with cell boundary pixels and in contact with cells listed the last argument of the function call

7. `secreteInsideCellAtCOM` – secretion at the center of mass of the cell

and 6 uptake modes:

1. `uptakeInsideCell` – this is equivalent to uptake in every pixel belonging to a cell

2. `uptakeInsideCellAtBoundary` – uptake takes place in the pixels belonging to the cell boundary

3. `uptakeInsideCellAtBoundaryOnContactWith` - uptake takes place in the pixels belonging to the cell boundary that touches any of the cells listed as the last argument of the function call

4. `uptakeOutsideCellAtBoundary` – uptake takes place in pixels which are outside the cell but in contact with cell boundary pixels

5. `uptakeOutsideCellAtBoundaryOnContactWith` - uptake takes place in pixels which are outside the cell but in contact with cell boundary pixels and in contact with cells listed the last argument of the function call

6. `uptakeInsideCellAtCOM` – uptake at the center of mass of the cell

Secretion functions use the following syntax:

```
secrete*(cell,amount,list_of_cell_types)
```

Note: The `list_of_cell_types` is used only for function which implement such functionality *i.e.* `secreteInsideCellAtBoundaryOnContactWith` and `secreteOutsideCellAtBoundaryOnContactWith`

Uptake functions use the following syntax:

```
uptake*(cell,max_amount,relative_uptake,list_of_cell_types)
```

Note: The `list_of_cell_types` is used only for function which implement such functionality *i.e.* `uptakeInsideCellAtBoundaryOnContactWith` and `uptakeOutsideCellAtBoundaryOnContactWith`

Note: Important: The uptake works as follows: when available concentration is greater than `max_amount`, then `max_amount` is subtracted from `current_concentration`, otherwise we subtract `relative_uptake*current_concentration`.

As you may infer from above, the modes 1-5 require tracking of pixels belonging to cell and pixels belonging to cell boundary. If you are not using those secretion modes you may disable pixel tracking by including:

```
<DisablePixelTracker/>
```

or

```
<DisableBoundaryPixelTracker/>
```

as shown in the example below:

```
<Plugin Name="Secretion">

    <DisablePixelTracker/>
    <DisableBoundaryPixelTracker/>

    <Field Name="ATTR" ExtraTimesPerMC="2">
        <Secretion Type="Bacterium">200</Secretion>
        <SecretionOnContact Type="Medium" SecreteOnContactWith="B">300</
↪SecretionOnContact>
        <ConstantConcentration Type="Bacterium">500</ConstantConcentration>
    </Field>
</Plugin>
```

Note: Make sure that fields into which you will be secreting chemicals exist. They are usually fields defined in PDE solvers. When using secretion plugin you do not need to specify SecretionData section for the PDE solvers.

When implementing e.g. secretion inside cell when the cell is in contact with other cell we use neighbor tracker and a short script in the spirit of the below snippet:

```
for cell in self.cellList:
    attrSecretor = self.getFieldSecretor("ATTR")
    for neighbor, commonSurfaceArea in self.getCellNeighborDataList(cell):
        if neighbor.type in [self.WALL]:
            attrSecretor.secreteInsideCell(cell, 300)
```

5.18 Secretion Plugin (legacy version)

Warning: While we still support Secretion plugin as described in this section we observed performance degradation when when declaring <Field> elements inside the plugin. To resolve this issue we encourage users to implement secretion "by cell type" in the PDE solver and keep using secretion plugin to implement secretion on a per-cell basis using Python scripting.

Note: In version 3.6.2 Secretion plugin should not be used with DiffusionSolverFE or any of the GPU-based solvers.

In earlier version os of CC3D secretion was part of PDE solvers. We still support this mode of model description however, starting in 3.5.0 we developed separate plugin which handles secretion only. Via secretion plugin we can simulate cellular secretion of various chemicals. The secretion plugin allows users to specify various secretion modes in the CC3DML file – CC3DML syntax is practically identical to the SecretionData syntax of PDE solvers. In addition to this Secretion plugin allows users to manipulate secretion properties of individual cells from Python level. To account for possibility of PDE solver being called multiple times during each MCS, the Secretion plugin can be

called multiple times in each MCS as well. We leave it up to user the rescaling of secretion constants when using multiple secretion calls in each MCS.

Note: Secretion for individual cells invoked via Python will be called only once per MCS.

Typical CC3DML syntax for Secretion plugin is presented below:

```
<Plugin Name="Secretion">
    <Field Name="ATTR" ExtraTimesPerMC="2">
        <Secretion Type="Bacterium">200</Secretion>
        <SecretionOnContact Type="Medium" SecreteOnContactWith="B">300</
→SecretionOnContact>
        <ConstantConcentration Type="Bacterium">500</ConstantConcentration>
    </Field>
</Plugin>
```

By default `ExtraTimesPerMC` is set to `0` - meaning no extra calls to `Secretion` plugin per MCS.

Typical use of secretion from Python is demonstrated best in the example below:

```
class SecretionSteppable(SecretionBasePy):
    def __init__(self, _simulator, _frequency=1):
        SecretionBasePy.__init__(self, _simulator, _frequency)

    def step(self, mcs):
        attrSecretor = self.getFieldSecretor("ATTR")
        for cell in self.cellList:
            if cell.type == 3:
                attrSecretor.secreteInsideCell(cell, 300)
                attrSecretor.secreteInsideCellAtBoundary(cell, 300)
                attrSecretor.secreteOutsideCellAtBoundary(cell, 500)
                attrSecretor.secreteInsideCellAtCOM(cell, 300)
```

5.19 PDESolverCaller Plugin

Warning: In most cases you can specify extra calls to PDE solvers in the solver itself. Thus this plugin is being deprecated. We mention it here for backward compatibility reasons as some of the older CC3D simulations may still be using this plugin.

PDE solvers in CompuCell3D are implemented as steppables . This means that by default they are called every MCS. In many cases this is insufficient. For example if diffusion constant is large, then explicit finite difference method will become unstable and the numerical solution will have no sense. To fix this problem one could call PDE solver many times during single MCS. This is precisely the task taken care of by `PDESolverCaller plugin`. The syntax is straightforward:

```
<Plugin Name="PDESolverCaller">
    <CallPDE PDESolverName="FlexibleDiffusionSolverFE"ExtraTimesPerMC="8"/>
</Plugin>
```

All you need to do is to give the name of the steppable that implements a given PDE solver and pass let CompCell3D know how many extra times per MCS this solver is to be called (here `FlexibleDiffusionSolverFE` was 8 extra times per MCS).

5.20 FocalPointPlasticity Plugin

`FocalPointPlasticity` puts constrains on the distance between cells' center of masses. A key feature of this plugin is that the list of "focal point plasticity neighbors" can change as the simulation evolves and user has to specifies the maximum number of "focal point plasticity neighbors" a given cell can have. Let's look at relatively simple CC3DML syntax of `FocalPointPlasticityPlugin` (see *Demos/PluginDemos/FocalPointPlasticity/FocalPointPlasticity* example and we will show more complex examples later):

```
<Plugin Name="FocalPointPlasticity">
   <Parameters Type1="Condensing" Type2="NonCondensing">
      <Lambda>10.0</Lambda>
      <ActivationEnergy>-50.0</ActivationEnergy>
      <TargetDistance>7</TargetDistance>
      <MaxDistance>20.0</MaxDistance>
      <MaxNumberOfJunctions>2</MaxNumberOfJunctions>
   </Parameters>

   <Parameters Type1="Condensing" Type2="Condensing">
      <Lambda>10.0</Lambda>
      <ActivationEnergy>-50.0</ActivationEnergy>
      <TargetDistance>7</TargetDistance>
      <MaxDistance>20.0</MaxDistance>
      <MaxNumberOfJunctions>2</MaxNumberOfJunctions>
   </Parameters>
   <NeighborOrder>1</NeighborOrder>
</Plugin>
```

`Parameters` section describes properties of links between cells. `MaxNumberOfJunctions`, `ActivationEnergy`, `MaxDistance` and `NeighborOrder` are responsible for establishing connections between cells. CC3D constantly monitors pixel copies and during pixel copy between two neighboring cells/subcells it checks if those cells are already participating in focal point plasticity constraint. If they are not, CC3D will check if connection can be made (e.g. `Condensing` cells can have up to two connections with `Condensing` cells and up to 2 connections with `NonCondensing` cells – see first line of `Parameters` section and `MaxNumberOfJunctions` tag). The `NeighborOrder` parameter determines the pixel vicinity of the pixel that is about to be overwritten which CC3D will scan in search of the new link between cells. `NeighborOrder 1` (which is default value if you do not specify this parameter) means that only nearest pixel neighbors will be visited. The `ActivationEnergy` parameter is added to overall energy in order to increase the odds of pixel copy which would lead to new connection.

Once cells are linked the energy calculation is carried out according to the formula:

$$E = \sum_{i,j,cell\ neighbors} \lambda_{ij} \left(l_{ij} - L_{ij}\right)^2 \tag{5.9}$$

where l_{ij} is a distance between center of masses of cells `i` and `j` and L_{ij} is a target length corresponding to l_{ij}.

λ_{ij} and L_{ij} between different cell types are specified using `Lambda` and `TargetDistance` tags. The `MaxDistance` determines the distance between cells' center of masses past which the link between those cells break. When the link breaks, then in order for the two cells to reconnect they would need to come in contact again. However it is usually more likely that there will be other cells in the vicinity of separated cells so it is more likely to establish new link than restore broken one.

The above example was one of the simplest examples of use of `FocalPointPlasticity`. A more complicated one involves compartmental cells. In this case each cell has separate "internal" list of links between cells belonging to the same cluster and another list between cells belonging to different clusters. The energy contributions from both lists are summed up and everything that we have said when discussing example above applies to compartmental cells. Sample syntax of the `FocalPointPlasticity` plugin which includes compartmental cells is shown

below. We use `InternalParameters` tag/section to describe links between cells of the same cluster (see *Demos/PluginDemos/FocalPointPlasticity/FocalPointPlasticityCompartments* example):

```
<Plugin Name="FocalPointPlasticity">

    <Parameters Type1="Top" Type2="Top">
        <Lambda>10.0</Lambda>
        <ActivationEnergy>-50.0</ActivationEnergy>
        <TargetDistance>7</TargetDistance>
        <MaxDistance>20.0</MaxDistance>
        <MaxNumberOfJunctions NeighborOrder="1">1</MaxNumberOfJunctions>
    </Parameters>

    <Parameters Type1="Bottom" Type2="Bottom">
        <Lambda>10.0</Lambda>
        <ActivationEnergy>-50.0</ActivationEnergy>
        <TargetDistance>7</TargetDistance>
        <MaxDistance>20.0</MaxDistance>
        <MaxNumberOfJunctions NeighborOrder="1">1</MaxNumberOfJunctions>
    </Parameters>

    <InternalParameters Type1="Top" Type2="Center">
        <Lambda>10.0</Lambda>
        <ActivationEnergy>-50.0</ActivationEnergy>
        <TargetDistance>7</TargetDistance>
        <MaxDistance>20.0</MaxDistance>
        <MaxNumberOfJunctions>1</MaxNumberOfJunctions>
    </InternalParameters>

    <InternalParameters Type1="Bottom" Type2="Center">
        <Lambda>10.0</Lambda>
        <ActivationEnergy>-50.0</ActivationEnergy>
        <TargetDistance>7</TargetDistance>
        <MaxDistance>20.0</MaxDistance>
        <MaxNumberOfJunctions>1</MaxNumberOfJunctions>
    </InternalParameters>

    <NeighborOrder>1</NeighborOrder>

</Plugin>
```

We can also specify link constituent law and change it to different form that "spring relation". To do this we use the following syntax inside FocalPointPlasticity CC3DML plugin:

```
<LinkConstituentLaw>
    <!--The following variables lare defined by default: Lambda,Length,TargetLength-->

    <Variable Name='LambdaExtra' Value='1.0'/>
    <Formula>LambdaExtra*Lambda*(Length-TargetLength)^2</Formula>

</LinkConstituentLaw>
```

By default CC3D defines 3 variables (`Lambda`, `Length`, `TargetLength`) which correspond to λ_{ij} , l_{ij} and L_{ij} from the formula above. We can also define extra variables in the CC3DML (e.g. `LambdaExtra`). The actual link constituent law obeys `muParser` syntax convention. Once link constituent law is defined it is applied to all focal point plasticity links. The example demonstrating the use of custom link constituent law can be found in *Demos/PluginDemos/FocalPointPlasticityCustom*.

Sometimes it is necessary to modify link parameters individually for every cell pair. In this case

we would manipulate `FocalPointPlasticity` links using Python scripting. Example *Demos/PluginDemos/FocalPointPlasticity/FocalPointPlasticityCompartments* demonstrates exactly this situation. You still need to include CC3DML section as the one shown above for compartmental cells, because we need to tell CC3D how to link cells. The only notable difference is that in the CC3DML we have to include `<Local/>` tag to signal that we will set link parameters (`Lambda`, `TargetDistance`, `MaxDistance`) individually for each cell pair:

```
<Plugin Name="FocalPointPlasticity">
    <Local/>
    <Parameters Type1="Top" Type2="Top">
        <Lambda>10.0</Lambda>
        <ActivationEnergy>-50.0</ActivationEnergy>
        <TargetDistance>7</TargetDistance>
        <MaxDistance>20.0</MaxDistance>
        <MaxNumberOfJunctions NeighborOrder="1">1</MaxNumberOfJunctions>
    </Parameters>
    ...
</Plugin>
```

Python steppable where we manipulate cell-cell focal point plasticity link properties is shown below:

```
class FocalPointPlasticityCompartmentsParams(SteppablePy):
    def __init__(self, _simulator, _frequency=10):
        SteppablePy.__init__(self, _frequency)
        self.simulator = _simulator
        self.focalPointPlasticityPlugin = CompuCell.getFocalPointPlasticityPlugin()
        self.inventory = self.simulator.getPotts().getCellInventory()
        self.cellList = CellList(self.inventory)

    def step(self, mcs):
        for cell in self.cellList:
            for fppd in InternalFocalPointPlasticityDataList(self.
→focalPointPlasticityPlugin, cell):
                self.focalPointPlasticityPlugin.
→setInternalFocalPointPlasticityParameters(cell, fppd.neighborAddress,
                                                                         ␣
→    0.0, 0.0, 0.0)
```

The syntax to change focal point plasticity parameters (or as here internal parameters) is as follows:

```
setFocalPointPlasticityParameters(cell1, cell2, lambda, targetDistance, maxDistance)
```

```
setInternalFocalPointPlasticityParameters(cell1, cell2, lambda, targetDistance,␣
→maxDistance)
```

Similarly, to inspect current values of the focal point plasticity parameters we would use the following Python construct:

```
for cell in self.cellList:
    for fppd in InternalFocalPointPlasticityDataList(self.focalPointPlasticityPlugin,␣
→cell):
        print "fppd.neighborId", fppd.neighborAddress.id
        " lambda=", fppd.lambdaDistance
```

For non-internal parameters we simply use `FocalPointPlasticityDataList` instead of `InternalFocalPointPlasticityDataList`.

Examples *Demos/PluginDemos/FocalPointPlasticity...* show in relatively simple way how to use

`FocalPointPlasticity` plugin. Those examples also contain useful comments.

Note: When using `FocalPointPlasticity` Plugin from `Mitosis` module one might need to break or create focal point plasticity links. To do so `FocalPointPlasticity` Plugin provides 4 convenience functions which can be invoked from the Python level:

```
deleteFocalPointPlasticityLink(cell1, cell2)

deleteInternalFocalPointPlasticityLink(cell1, cell2)

createFocalPointPlasticityLink(cell1, cell2, lambda , targetDistance, maxDistance)

createInternalFocalPointPlasticityLink(cell1, cell2, lambda , targetDistance,
→maxDistance)
```

5.21 Curvature Plugin

The `Curvature` plugin implements energy term for compartmental cells. The mathematics and mechanics between the plugin have been described in "A New Mechanism for Collective Migration in *Myxococcus xanthus*", J. Starruß, Th. Bley, L. Søgaard-Andersen and A. Deutsch, *Journal of Statistical Physics*, DOI: **10.1007/s10955-007-9298-9**, (2007). For a "long" compartmental cell composed of many subcells (think of a snake-like elongated sequence of compartments) it imposes a constraint on curvature of cells. The syntax is slightly complex:

```xml
<Plugin Name="Curvature">

   <InternalParameters Type1="Top" Type2="Center">
      <Lambda>100.0</Lambda>
      <ActivationEnergy>-50.0</ActivationEnergy>
   </InternalParameters>

   <InternalParameters Type1="Center" Type2="Center">
      <Lambda>100.0</Lambda>
      <ActivationEnergy>-50.0</ActivationEnergy>
   </InternalParameters>

   <InternalParameters Type1="Bottom" Type2="Center">
      <Lambda>100.0</Lambda>
      <ActivationEnergy>-50.0</ActivationEnergy>
   </InternalParameters>

   <InternalTypeSpecificParameters>
      <Parameters TypeName="Top" MaxNumberOfJunctions="1" NeighborOrder="1"/>
      <Parameters TypeName="Center" MaxNumberOfJunctions="2" NeighborOrder="1"/>
      <Parameters TypeName="Bottom" MaxNumberOfJunctions="1" NeighborOrder="1"/>
   </InternalTypeSpecificParameters>

</Plugin>
```

The `InternalTypeSpecificParameter` informs `Curvature` Plugin how many neighbors a cell of given type will have. In the case of "snake-shaped" cell the numbers that make sense are 1 and 2. The middle segment will have 2 connections and `head` and `tail` segments will have only one connection with neighboring segments (subcells). The connections are established dynamically. The way it happens is that during simulation CC3D constantly monitors pixel copies and during pixel copy between two neighboring cells/subcells it checks if those cells are already "connected"

---

using curvature constraint. If they are not, CC3D will check if connection can be made (e.g. `Center` cells can have up to two connections and `Top` and `Bottom` only one connection). Usually establishing connections takes place at the beginning of the simulation and often happens within first Monte Carlo Step (depending on actual initial configuration, of course, but if segments touch each other connections are established almost immediately). The `ActivationEnergy` parameter is added to overall energy in order to increase the odds of pixel copy which would lead to new connection. `Lambda` tag/parameter determines "the strength" of curvature constraint. The higher the `Lambda` the more stiffer cells will be *i.e.* they will tend to align along straight line.

5.22 BoundaryPixelTracker Plugin

`BoundaryPixelTracker` plugin keeps a list of boundary pixels for each cell. The syntax is as follows:

```
<Plugin Name="BoundaryPixelTracker">
    <NeighborOrder>1</NeighborOrder>
</Plugin>
```

This plugin is also used by other plugins as a helper module. Examples use of this plugin is found in *Demos/PluginDemos/BoundaryPixelTracker_xxx*.

5.23 GlobalBoundaryPixelTracker Plugin

`GlobalBoundaryPixelTracker` plugin tracks boundary pixels of **all** the cells including `Medium`. It is used in a `Boundary Walker` algorithm where instead of blindly picking pixel copy candidate we pick it from the set of pixels comprising boundaries of non frozen cells. In situations when lattice is large and there are not that many cells it makes sense to use `BoundaryWalker` algorithm to limit number of pixel picks that would not lead to actual pixel copy.

Note: `BoundaryWalkerAlgorithm` does not really work with OpenMP version of CC3D which includes all versions starting with `3.6.0`.

Take a look at the following example:

```
<Potts>
   <Dimensions x="100" y="100" z="1"/>
   <Anneal>10</Anneal>
   <Steps>10000</Steps>
   <Temperature>5</Temperature>
   <Flip2DimRatio>1</Flip2DimRatio>
   <NeighborOrder>2</NeighborOrder>
   <MetropolisAlgorithm>BoundaryWalker</MetropolisAlgorithm>
   <Boundary_x>Periodic</Boundary_x>
 </Potts>

 <Plugin Name="GlobalBoundaryPixelTracker">
    <NeighborOrder>2</NeighborOrder>
 </Plugin>
```

Here we are using `BoundaryWalker` algorithm (`Potts` section) and subsequently we list `GlobalBoundaryTracker` plugin where we set neighbor order to match that in the Potts section. The neighbor order determines how "thick" the overall boundary of cells will be. The higher this number the more pixels will belong to the boundary.

5.24 PixelTracker Plugin

`PixelTracker` plugin allows storing list of all pixels belonging to a given cell. The syntax is as follows:

```
<Plugin Name="PixelTracker">
    <TrackMedium/>
</Plugin>
```

This plugin is also used by other modules (e.g. `Mitosis`) as a helper module. Simple example can be found in *Demos/PluginDemos/PixelTrackerExample*. Beginning with `4.1.2`, medium pixels can be optionally tracked using `TrackMedium`. This feature is automatically enabled by attaching a Fluctuation Compensator to a PDE solver.

5.25 MomentOfInertia Plugin

`MomentOfInertia` plugin keeps up-to-date tensor of inertia for every cell. Internally, it uses parallel axis theorem to calculate most up-to-date tensor of inertia. While, the plugin can be called directly:

```
<Plugin Name="MomentOfInertia"/>
```

most commonly it is called indirectly by other plugins like e.g. `LengthConstraint` plugin.

`MomentOfInertia` plugin gives users access (via Python scripting) to current lengths of cell's semiaxes. Examples in *Demos/PluginDemos/MomentOfInertia* demonstrate how to get lengths of semiaxes. For example, to get semiaxes lengths for a given `cell`, in Python we would type:

```
axes=self.momentOfInertiaPlugin.getSemiaxes(cell)
```

`axes` is a 3-component vector with 0th element being length of minor axis, 1st – length of median axis (which is set to 0 in 2D) and 2nd element indicating the length of major semiaxis.

Note: Important: Because calculating lengths of semiaxes involves quite a few of floating point operations it may happen (usually on hexagonal lattice) that for cells composed of 1, 2, or 3 pixels one moment the square of one of the semiaxes may end up being slightly negative leading to `NaN` (not a number) length. This is due to round-off error and whenever CC3D detects very small absolute value of square of the length of semiaxes (10^{-6}) it sets length of this semiaxes to `0.0` regardless whether the squared value is positive or negative. However, it is a good practice to test whether the length of semiaxis is sane by adding a simple `if` statement as shown below (here we show how to test for a `NaN`):

```
if length != length:
    print "length is NaN":
else:
    print "length is a proper floating point number"
```

5.26 ConvergentExtension plugin

Note: This is very specialized plugin which currently is in Tier 2 plugins in terms of support. It attempts to implement energy term described in "Simulating Convergent Extension by Way of Anisotropic Differemtial Adhesion", *Zajac* M,

---

Jones GL, and *Glazier* JA, Journal of Theoretical Biology **222** (2), 2003. However due to certain ambiguities in the plugin description we had difficulties to getting it to work properly.

Note: A better way to implement convergent extension is to follow the simulations described in "Filopodial-Tension Model of Convergent-Extension of Tissues", *Julio M. Belmonte* , *Maciej H. Swat*, *James A. Glazier*, PLoS Comp Bio https://doi.org/10.1371/journal.pcbi.1004952

`ConvergentExtension` plugin presented here is a somewhat simplified version of energy term described *Mark Zajac's* paper.

This plugin uses the following syntax:

The `Alpha` tag represents numerical value of α parameter from the paper.

Steppable Section

Steppables are CompuCell modules that are called every Monte Carlo Step (MCS). More precisely, they are called after all the pixel copy attempts in a given MCS have been carried out. Steppables may have various functions - for example solving PDEs, checking if critical concentration threshold have been reached, updating target volume or target surface given the concentration of come growth factor, initializing cell field, writing numerical results to a file, *etc...* In general, steppables perform all functions that need to be done every MCS. In the reminder of this section we will present steppables currently available in the CompuCell3D and describe their usage.

Tip: It is most convenient to implement Steppables in Python. However, in certain situations where code performance is an issue users can implement steppables in C++

This section "off-the-shelf" steppables that are available in CC3D and were implemented using C++

- uniform_initializer
- blob_initializer
- pif_initializer
- pif_dumper
- mitosis
- box_watcher
- mu-parser

6.1 BoxWatcher Steppable

Warning: Functionality of this module has been reduced in CC3D versions that support parallel computations (`3.6.0` and up). Main motivation for this module was to speed up computations but with parallel version the need for this module is somewhat smaller.

This steppable can potentially speed-up your simulation. Every MCS (or every `Frequency` MCS) it determines maximum and minimum coordinates of cells and then imposes slightly bigger box around cells and ensures that in the subsequent MCS pixel copy attempts take place only inside this box containing cells (plus some amount of medium on the sides). Thus, instead of sweeping entire lattice and attempting random pixel copies CompuCell3D will only spend time trying flips inside the box. Depending on the simulation the performance gains are up to approx. 30%. The steppable will work best if you have simulation with cells localized in one region of the lattice with lots of empty space. The steppable will adjust box every MCS (or every `Frequency` MCS) according to evolving cellular pattern.

The syntax is as follows:

All that is required is to specify amount of extra space (expressed in units of pixels) that needs to be added to a tight box i.e. the box whose sides just touch most peripheral cells' pixels.

6.2 BlobInitializer Steppable

`BlobInitializer` steppable is used to lay out circular blob of cells on the lattice.

An example syntax where we create one circular region of cells in the lattice is presented below:

```
<Steppable Type="BlobInitializer">
   <Region>
     <Gap>0</Gap>
     <Width>5</Width>
     <Radius>40</Radius>
     <Center x="100" y="100" z="0"/>
     <Types>Condensing,NonCondensing</Types>
   </Region>
</Steppable>
```

Similarly as for the `UniformFieldInitializer` users can define many regions each of which is a blob of a particular center point, radius and list of cell types that will be assigned to cells forming the blob. Listing types in the `<Types>` tag follows same rules as in the UniformInitializer.

Note: Original (**and deprecated**) syntax of this plugin looks as follows:

```
<Steppable Type="BlobInitializer">
    <Gap>0</Gap>
    <Width>5</Width>
    <CellSortInit>yes</CellSortInit>
    <Radius>40</Radius>
</Steppable>
```

The blob is centered in the middle of th lattice and has radius given by `<Radius>` parameter. All cells are initially squares (or cubes in 3D) where `<Width>` determines the length of the cube or square side and `<Gap>` determines space between squares or cubes. `<CellSortInit>` tag and value `yes` is used to initialize cells randomly with type id being either `1` or `2` . Otherwise all cells will have type id `1`. This can be easily modified in Python .

6.3 PIF Initializer

To initialize the configuration of the simulation lattice we can write custom **lattice initialization file**. Our experience suggests that you will probably have to write your own initialization files rather than relying on built-in initializers. The reason is simple: the built-in initializers implement very simple cell layouts, and if you want to study more

complicated cell arrangements, the built-in initializers will not be very helpful. Therefore, we encourage you to learn how to prepare lattice initialization files. We have developed `CellDraw` tool which is a part of CC3D suite and it allows users to draw initial cell layout in a very intuitive way. We encourage you to read "Introduction to CellDraw" to familiarize yourself with this tool.

To import custom cell layouts, CompuCell3D uses very simple **Potts Initial File** `PIF` file format. It tells CompuCell3D how to lay out assign the simulation lattice pixels to cells.

The PIF consists of multiple lines of the following format:

```
cell# celltype x1 x2 y1 y2 z1 z2
```

Where `cell#` is the unique integer index of a cell, `celltype` is a string representing the cell's initial type, and `x1` and `x2` specify a *range* of x-coordinates contained in the cell (similarly `y1` and `y2` specify a range of y-coordinates and `z1` and `z2` specify a range of z-coordinates). Thus each line assigns a rectangular volume to a cell. If a cell is not perfectly rectangular, multiple lines can be used to build up the cell out of rectangular sub-volumes (just by reusing the `cell#` and `celltype`).

A PIF can be provided to CompuCell3D by including the steppable object **PIFInitializer**

Let's look at a PIF example for foams:

```
0 Medium 0 101 0 101 0 0
1 Foam 13 25 0 5 0 0
2 Foam 25 39 0 5 0 0
3 Foam 39 46 0 5 0 0
4 Foam 46 57 0 5 0 0
5 Foam 57 65 0 5 0 0
6 Foam 65 76 0 5 0 0
7 Foam 76 89 0 5 0 0
```

These lines define a background of `Medium` which fills the whole lattice and is then overwritten by seven rectangular cells of type `Foam` numbered `1` through `7`. Notice that these cells lie in the `xy` plane (`z1=0 z2=0` implies that cells have thickness =1) so this example is a two-dimensional initialization.

You can write the PIF file manually, but using a script or program that will write PIF file for you in the language of your choice (Perl, Python, Matlab, Mathematica, C, C++, Java or any other programming language) will save a great deal of typing.

Notice, that for compartmental cell model the format of the PIF file is different:

```
Include Clusters
cluster # cell# celltype x1 x2 y1 y2 z1 z2
```

For example:

```
Include Clusters
1 1 Side1 23 25 47 56 10 14
1 2 Center 26 30 50 54 10 14
1 3 Side2 31 33 47 56 10 14
1 4 Top 26 30 55 59 10 14
1 5 Bottom 26 30 45 49 10 14
2 6 Side1 35 37 47 56 10 14
2 7 Center 38 42 50 54 10 14
2 8 Side2 43 45 47 56 10 14
2 9 Top 38 42 55 59 10 14
2 10 Bottom 38 42 45 49 10 14
```

Tip: An easy way to generate PIF file from the current simulation snapshot is to use Player `Tools->Generate` PIF file from current snapshot. . . menu option. Alternatively we can use PIFDumper steppable discussed next.

6.4 PIFDumper Steppable

This steppable does the opposite to `PIFIitialize``r – it writes PIF file of current lattice configuration. The syntax similar to the syntax of ``PIFInitializer:`

```
<Steppable Type="PIFDumper" Frequency="100">
    <PIFName>line</PIFName>
</Steppable>
```

Notice that we used `Frequency` attribute of steppable to ensure that PIF files are written every `100` MCS. Without it they would be written every MCS. The file names will have the following format: `PIFName.MCS.pif`

In our case they would be `line.0.pif`, `line.100.pif`, `line.200.pif`, etc. . .

This module is actually quite useful. For example, if we want to start simulation from a more configuration of cells (not rectangular cells as this is the case when we use `Uniform` or `Blob` initializers). In such a case we would run a simulation with a `PIFDumper` included and once the cell configuration reaches desired shape we would stop and use PIF file corresponding to this state. Once we have PIF initial configuration we may run many simulation starting from the same, realistic initial condition.

Tip: Restarting simulation from a given configuration is actually even easier in the recent versions of CC3D. All you have to do is to create .``cc3d`` project where you add serialization optyion CC3D will be savbing complete snapshots of the simulation (including PIF files) and you can easily restart the simulation from a given end-point of the previous run. For more details see "Python Scripting Manual" https://pythonscriptingmanual.readthedocs.io/en/latest/restarting_simulations.html?highlight=restart

---

Tip: You can also generate PIF file from the current simulation snapshot by using Player tool: Tools->Generate PIF file from current snapshot. . .

6.5 Mitosis Steppable.

Mitosis steppable is described in great detail in "Python Scripting Manual" - see for example https://pythonscriptingmanual.readthedocs.io/en/latest/mitosis.html?highlight=mitosis

but because of its importance we are including a copy of that description here.

In developmental simulations we often need to simulate cells which grow and divide. In earlier versions of Compu-Cell3D we had to write quite complicated plugin to do that which was quite cumbersome and unintuitive. The only advantage of the plugin was that mitosis was taking place immediately after the pixel copy which had triggered mitosis condition. This guaranteed that any cell which was supposed divide at any instance in the simulation, actually did. However, because state of the simulation is normally observed after completion of full a Monte Carlo Step, and not in the middle of MCS it makes actually more sense to implement `Mitosis` as a steppable. Let us examine the simplest simulation which involves mitosis. We start with a single cell and grow it. When cell reaches critical (doubling) volume it divides. We check if the cell has reached doubling volume at the end of each MCS. The folder containing this simulation is

---

Demos/CompuCellPythonTutorial/steppableBasedMitosis. The mitosis algorithm is implemented in *Demos/CompuCellPythonTutorial/steppableBasedMitosis/Simulation/steppableBasedMitosisSteppables.py*

File:

Demos/CompuCellPythonTutorial/steppableBasedMitosis/Simulation/steppableBasedMitosisSteppables.py

```python
from PySteppables import *
from PySteppablesExamples import MitosisSteppableBase
import CompuCell

class VolumeParamSteppable(SteppableBasePy):
    def __init__(self, _simulator, _frequency=1):
        SteppableBasePy.__init__(self, _simulator, _frequency)

    def start(self):
        for cell in self.cellList:
            cell.targetVolume = 25
            cell.lambdaVolume = 2.0

    def step(self, mcs):
        for cell in self.cellList:
            cell.targetVolume += 1

class MitosisSteppable(MitosisSteppableBase):
    def __init__(self, _simulator, _frequency=1):
        MitosisSteppableBase.__init__(self, _simulator, _frequency)

        # 0 - parent child position will be randomized between mitosis event
        # negative integer - parent appears on the 'left' of the child
        # positive integer - parent appears on the 'right' of the child
        self.setParentChildPositionFlag(-1)

    def step(self, mcs):
        cells_to_divide = []
        for cell in self.cellList:
            if cell.volume > 50:
                cells_to_divide.append(cell)

        for cell in cells_to_divide:
            # to change mitosis mode leave one of the below lines uncommented
            self.divideCellRandomOrientation(cell)

    def updateAttributes(self):
        self.parentCell.targetVolume /= 2.0  # reducing parent target volume
        self.cloneParent2Child()

        if self.parentCell.type == self.CONDENSING:
            self.childCell.type = self.NONCONDENSING
        else:
            self.childCell.type = self.CONDENSING
```

Two steppables:`` VolumeParamSteppable`` and MitosisSteppable are the essence of the above simulation. The first steppable initializes volume constraint for all the cells present at T=0 MCS (only one cell) and then every 10 MCS (see the frequency with which VolumeParamSteppable in initialized to run - *Demos/CompuCellPythonTutorial/steppableBasedMitosis/Simulation/steppableBasedMitosis.py*) it increases target volume of cells, effectively causing cells to grow.

```
from steppableBasedMitosisSteppables import VolumeParamSteppable
volumeParamSteppable=VolumeParamSteppable(sim ,10)
steppableRegistry.registerSteppable(volumeParamSteppable)

from steppableBasedMitosisSteppables import MitosisSteppable
mitosisSteppable=MitosisSteppable(sim, 10)
steppableRegistry.registerSteppable(mitosisSteppable)
```

The second steppable checks every `10` MCS (we can, of course, run it every MCS) if cell has reached doubling volume of `50`. If it did such cell is added to the list cells_to_divide. After construction of `cells_to_divide` is complete we iterate over this list and divide all the cells in it.

> **Warning:** It is important to divide cells outside the loop where we iterate over entire cell inventory. If we keep dividing cells in this loop we are adding elements to the list over which we iterate over and this might have unwanted side effects. The solution is to use use list of cells to divide as we did in the example.

Notice that we call `self.divideCellRandomOrientation(cell)` function to divide cells. Other modes of division are available as well and they are as follows:

```
self.divideCellOrientationVectorBased(cell,1,0,0)
self.divideCellAlongMajorAxis(cell)
self.divideCellAlongMinorAxis(cell)
```

Notice that `MitosisSteppable` inherits `MitosisSteppableBase` class (defined in `PySteppablesExamples.py`).It is the base class which ensures that after we call any of the cell dividing function (e.g. `divideCellRandomOrientation`) CompuCell3D will automatically call `updateAttributes` function as well. `updateAttributes` function is very important and we must call it in order to ensure integrity and sanity of the simulation. During mitosis a new cell is created (accessed in Python as childCell – defined in `MitosisSteppableBase` - `self.mitosisSteppable.childCell`) and as such this cell is uninitialized. It does have default attributes (read-only) of a cell such as volume, surface (if we decide to use surface constraint or `SurfaceTracker` plugin) but all other parameters of such cell are set to default values. In our simulation we have been setting `targetVolume` and `lambdaVolume` individually for each cell. After mitosis `childCell` will need those parameters to be set as well. To make things more interesting, in our simulation we decided to change type of cell to be different than type of parent cell. In more complex simulations where cells have more attributes which are used in the simulation, we have to make sure that in the `updateAttributes` function `childCell` and its attributes get properly initialized. It is also very common practice to change attributes of parentCell after mitosis as well to account for the fact that parentCell is not the original parentCell from before the mitosis.

> **Note:** If you specify orientation vector for the mitosis the actual division will take place along the line/plane **perpendicular to this vector**.

> **Note:** The name of the function where we update attributes after mitosis has to be exactly `updateAtttributes`. If it is called differently CC3D will not call it automatically. We can of course call such function by hand, immediately we do the mitosis but this is not very elegant solution.

6.6 muParser

CC3D uses `muParser` to allow users specify simple mathematical expressions in the XML (or XML-equivalent Python scripts). The following link points to full specification of the `muParser`: http://muparser.sourceforge.net/mup_features.html#idDef2. The general guideline to using muParser syntax inside XML is to enclose `muParser` expression between `<![CDATA[` and `]]>`:

```
<XML_ELEMENT_WITH_MUPARSER_EXPRESSION>
    <![CDATA[
        MUPARSER EXPRESSION
    ]]>
</XML_ELEMENT_WITH_MUPARSER_EXPRESSION>
```

For example:

```
<AdditionalTerm>
    <![CDATA[
        CellType<1 ? 0.01*F : 0.15*F
    ]]>
</AdditionalTerm>
```

The reason for enclosing `muParser` expression between `<![CDATA[` and `]]>` is to prevent XML parser from interpreting < or > as beginning or end of the XML elements

Alternatively you may replace XML with equivalent Python syntax in which case things will look a bit simpler:

```
DiffusionDataElmnt_2.ElementCC3D("AdditionalTerm",{}," CellType<1 ? 0.01*F : 0.15*F ")
```

PDESolvers in CompuCell3D

One of the most important and time consuming parts of the CC3D simulation is to solve all sorts of Partial Differential Equations which describe behavior of certain simulation objects (usually chemical fields). Most of the CC3D PDE solvers solve PDE with diffusive terms. Because we are dealing with moving boundary condition problems it was easiest and probably most practical to use explicit scheme of Finite Difference method. Most of CC3D PDE solvers run on multi core architectures and we also have GPU solvers which run and high performance GPU's and they also provide biggest speedups in terms of performance. Because CC3D solvers were implemented at different CC3D life cycle and often in response to particular user requests, CC3DML specification may differ from solver to solver. However, the basic structure of CC3DML PDE solver code follows similar pattern .

- flexible_diffusion_solver

- diffusion_solver_settings

- diffusion_solver

- advection_diffusion_solver

- fast_diffusion_solver_2D

- kernel_diffusion_solver

- reaction_diffusion_solver

- steady_state_diffusion_solver

- fluctuation_compensator_addon

7.1 FlexibleDiffusionSolver

This steppable is one of the basic and most important modules in CompuCell3D simulations.

Tip: Starting from version `3.6.2` we developed `DiffusionSolverFE` which eliminates several inconveniences of `FlexibleDiffusionSolver.`

As the name suggests it is responsible for solving diffusion equation but in addition to this it also handles chemical secretion which maybe thought of as being part of general diffusion equation:

$$\frac{\partial c}{\partial t} = D\nabla^2 c + kc + \text{secretion} \tag{7.1}$$

where `k` is a decay constant of concentration `c` and `D` is the diffusion constant. The term called `secretion` has the meaning as described below.

Example syntax for FlexibleDiffusionSolverFE looks as follows:

```
<Steppable Type="FlexibleDiffusionSolverFE">
    <AutoscaleDiffusion/>
    <DiffusionField Name="FGF8">
        <DiffusionData>
            <FieldName>FGF8</FieldName>
            <DiffusionConstant>0.1</DiffusionConstant>
            <DecayConstant>0.002</DecayConstant>
            <ExtraTimesPerMCS>5</ExtraTimesPerMCS>
            <DeltaT>0.1</DeltaT>
            <DeltaX>1.0</DeltaX>
            <DoNotDiffuseTo>Bacteria</DoNotDiffuseTo>
            <InitialConcentrationExpression>x*y
            </InitialConcentrationExpression>
        </DiffusionData>

        <SecretionData>
            <Secretion Type="Amoeba">0.1</Secretion>
        </SecretionData>

        <BoundaryConditions>
            <Plane Axis="X">
                <ConstantValue PlanePosition="Min" Value="10.0"/>
                <ConstantValue PlanePosition="Max"  Value="10.0"/>
            </Plane>

            <Plane Axis="Y">
                <ConstantDerivative PlanePosition="Min" Value="10.0"/>
                <ConstantDerivative PlanePosition="Max"  Value="10.0"/>
            </Plane>
        </BoundaryConditions>

    </DiffusionField>

    <DiffusionField Name="FGF">
        <DiffusionData>
            <FieldName>FGF</FieldName>
            <DiffusionConstant>0.02</DiffusionConstant>
            <DecayConstant>0.001</DecayConstant>
            <DeltaT>0.01</DeltaT>
            <DeltaX>0.1</DeltaX>
            <DoNotDiffuseTo>Bacteria</DoNotDiffuseTo>
        </DiffusionData>
        <SecretionData>
            <SecretionOnContact Type="Medium"
             SecreteOnContactWith="Amoeba">0.1</SecretionOnContact>
            <Secretion Type="Amoeba">0.1</Secretion>
        </SecretionData>
    </DiffusionField>
</Steppable>
```

We define sections that describe a field on which the steppable is to operate. In our case we declare just two diffusion fields - `FGF8` and `FGF`.

> **Warning:** When you want to solve more than one diffusive field using given diffusion you should declare those multiple fiuelds within a single definition of the diffusion solver. You cannot include two diffusion solver definitions - one with e.g. `FGF8` and another with `FGF`. In other words you can only define e.g. `FlexibleDiffusionSolverFE` once per simulation. You can however use FlexibleDiffusionSolverFE for e.g. `FGF` and `DiffusionSolverFE` for e.g. `FGF8`

Inside the diffusion field we specify sections describing diffusion and secretion. Let's take a look at `DiffusionData` section first:

```
<DiffusionField Name="FGF8">
<DiffusionData>
    <FieldName>FGF8</FieldName>
    <DiffusionConstant>0.1</DiffusionConstant>
    <DecayConstant>0.002</DecayConstant>
    <ExtraTimesPerMCS>5</ExtraTimesPerMCS>
    <DeltaT>0.1</DeltaT>
    <DeltaX>1.0</DeltaX>
    <DoNotDiffuseTo>Bacteria</DoNotDiffuseTo>

    <InitialConcentrationExpression>x*y
    </InitialConcentrationExpression>

</DiffusionData>
```

We give a name (`FGF8`) to the diffusion field – this is required as we will refer to this field in other modules.

Next we specify diffusion constant and decay constant.

> **Warning:** We use Forward Euler Method to solve these equations. This is not a stable method for solving diffusion equation and we do not perform stability checks. If you enter too high diffusion constant for example you may end up with unstable (wrong) solution. Always test your parameters to make sure you are not in the unstable region.

We may also specify cells that will not participate in the diffusion. You do it using `<DoNotDiffuseTo>` tag. In this example we do not let any `FGF` diffuse into `Bacteria` cells. You may of course use as many as necessary `<DoNotDiffuseTo>` tags. To prevent decay of a chemical in certain cells we use syntax:

```
<DoNotDecayIn>Medium</DoNotDecayIn>
```

In addition to diffusion parameters we may specify how secretion should proceed. `SecretionData` section contains all the necessary information to tell CompuCell how to handle secretion. Let's study the example:

```
<SecretionData>
    <SecretionOnContact Type="Medium" SecreteOnContactWith="Amoeba">0.1</
→SecretionOnContact>
    <Secretion Type="Amoeba">0.1</Secretion>
</SecretionData>
```

Here we have a definition two major secretion modes. Line:

```
<Secretion Type="Amoeba">0.1</Secretion>
```

ensures that every cell of type `Amoeba` will get `0.1` increase in concentration every MCS. Line:

```
<SecretionOnContact Type="Medium" SecreteOnContactWith="Amoeba">0.1</
↪SecretionOnContact>
```

means that cells of type `Medium` will get additional `0.1` increase in concentration **but only when** they touch cell of type `Amoeba`. This mode of secretion is called `SecretionOnContact`.

We can also see new CC3DML tags `<DeltaT>` and `<DeltaX>`. Their values determine the correspondence between MCS and actual time and between lattice spacing and actual spacing size. In this example for the first diffusion field one MCS corresponds to `0.``1 units of actual time and lattice spacing is equal ``1` unit of actual length. What is happening here is that the diffusion constant gets multiplied by:

```
DeltaT/(DeltaX * DeltaX)
```

provided the decay constant is set to 0. If the decay constant is not zero DeltaT appears additionally in the term (in the explicit numerical approximation of the diffusion equation solution) containing decay constant so in this case it is more than simple diffusion constant rescaling.

`DeltaT` and `DeltaX` settings are closely related to `ExtraTimesPerMCS` setting which allows calling of diffusion (and only diffusion) more than once per MCS. The number of extra calls per MCS is specified by the user on a per-field basis using `ExtraTimesPerMCS` tag.

> **Warning:** When using `ExtraTimesPerMCS` secretion functions will called only once per MCS. This is different than using `PDESolverCaller` where entire module is called multiple times (this include diffusion and secretion for all fields).

> **Tip:** We recommend that you stay away from redefining `DeltaX` and `DeltaT` and assume that your diffusion/decay coefficients are expressed in units of pixel (distance) and MCS (time). This way when you assign physical time and distance units to MCS and pixels you can easily obtain diffusion and decay constants. `DeltaX` and `DeltaT` introduce unnecessary complications.

The `AutoscaleDiffusion` tag tells CC3D to automatically rescale diffusion constant when switching between square and hex lattices. In previous versions of CC3D such scaling had to be done manually to ensure that solutions diffusion of equation on different lattices match. Here we introduced for user convenience a simple tag that does rescaling automatically. The rescaling factor comes from the fact that the discretization of the divergence term in the diffusion equation has factors such as unit lengths, using surface are and pixel/voxel volume in it. On a square lattice all those values have numerical value of `1.0`. On hex lattice, and for that matter of non-square latticeses, only pixel/voxel volume has numerical value of `1`. All other quantities have values different than `1.0` which causes the necessity to rescale diffusion constant. The detail of the hex lattice derivation will be presented in the "Introduction to Hexagonal Lattices in CompuCell3D" - http://www.compucell3d.org/BinDoc/cc3d_binaries/Manuals/HexagonalLattice.pdf .

The `FlexibleDiffusionSolver` is also capable of solving simple coupled diffusion type PDE of the form:

$$\frac{\partial c}{\partial t} = D\nabla^2 c + kc + \text{secretion} + m_d cd + m_f cf$$

$$\frac{\partial d}{\partial t} = D\nabla^2 d + kd + \text{secretion} + n_c dc + n_f df$$

$$\frac{\partial f}{\partial t} = D\nabla^2 f + kf + \text{secretion} + p_c fc + p_d fd$$

where $m_c, m_f, n_c, n_f, p_c, p_d$ are coupling coefficients. To code the above equations in CC3DML syntax you need to use the following syntax:

```
<Steppable Type="FlexibleDiffusionSolverFE">
    <DiffusionField>
        <DiffusionData>
            <FieldName>c</FieldName>
            <DiffusionConstant>0.1</DiffusionConstant>
            <DecayConstant>0.002</DecayConstant>
            <CouplingTerm InteractingFieldName="d" CouplingCoefficent="0.1"/>
            <CouplingTerm InteractingFieldName="f" CouplingCoefficent="0.2"/>
            <DeltaT>0.1</DeltaT>
            <DeltaX>1.0</DeltaX>
            <DoNotDiffuseTo>Bacteria</DoNotDiffuseTo>
        </DiffusionData>
        <SecretionData>
            <Secretion Type="Amoeba">0.1</Secretion>
        </SecretionData>
    </DiffusionField>

    <DiffusionField>
        <DiffusionData>
            <FieldName>d</FieldName>
            <DiffusionConstant>0.02</DiffusionConstant>
            <DecayConstant>0.001</DecayConstant>
            <CouplingTerm InteractingFieldName="c" CouplingCoefficent="-0.1"/>
            <CouplingTerm InteractingFieldName="f" CouplingCoefficent="-0.2"/>
            <DeltaT>0.01</DeltaT>
            <DeltaX>0.1</DeltaX>
            <DoNotDiffuseTo>Bacteria</DoNotDiffuseTo>
            </DiffusionData>
        <SecretionData>
            <Secretion Type="Amoeba">0.1</Secretion>
        </SecretionData>
    </DiffusionField>

    <DiffusionField>
        <DiffusionData>
            <FieldName>f</FieldName>
            <DiffusionConstant>0.02</DiffusionConstant>
            <DecayConstant>0.001</DecayConstant>
            <CouplingTerm InteractingFieldName="c" CouplingCoefficent="-0.2"/>
            <CouplingTerm InteractingFieldName="d" CouplingCoefficent="0.2"/>
            <DeltaT>0.01</DeltaT>
            <DeltaX>0.1</DeltaX>
            <DoNotDiffuseTo>Bacteria</DoNotDiffuseTo>
        </DiffusionData>
        <SecretionData>
            <Secretion Type="Amoeba">0.1</Secretion>
        </SecretionData>
    </DiffusionField>
</Steppable>
```

As one can see the only addition that is required to couple diffusion equations has simple syntax:

```
<CouplingTerm InteractingFieldName="c" CouplingCoefficent="-0.1"/>
<CouplingTerm InteractingFieldName="f" CouplingCoefficent="-0.2"/>
```

7.2 Instabilities of the Forward Euler Method

Most of the PDE solvers in CC3D use Forward Euler explicit numerical scheme. This method is unstable for large diffusion constant. As a matter of fact using D=0.25 with pulse initial condition will lead to instabilities in 2D. To deal with this you would normally use implicit solvers however due to moving boundary conditions that we have to deal with in CC3D simulations, memory requirements, performance and the fact that most diffusion constants encountered in biology are quite low (unfortunately this is not for all chemicals e.g. oxygen) we decided to use explicit scheme. If you have to use large diffusion constants with explicit solvers you need to do rescaling:

1) Set D, Δt, Δx according to your model

2) If

$$D\frac{\Delta t}{\Delta x^2} > 0.16 in 3D \tag{7.2}$$

you will need to to call solver multiple times per MCS.

3) Set `<ExtraTimesPerMCS>` to N-1 where:

$$ND' = D \tag{7.3}$$

and

$$D\frac{\Delta t/N}{\Delta x^2} < 0.16 in 3D \tag{7.4}$$

7.3 Initial Conditions

We can specify initial concentration as a function of x, y, z coordinates using `<InitialConcentrationExpression>` tag we use `muParser` syntax to type the expression. Alternatively we may use `ConcentrationFileName` tag to specify a text file that contains values of concentration for every pixel:

```
<ConcentrationFileName>initialConcentration2D.txt</ConcentrationFileName>
```

The value of concentration of the last pixel read for a given cell becomes an overall value of concentration for a cell. That is if cell has, say 8 pixels, and you specify different concentration at every pixel, then cell concentration will be the last one read from the file.

Concentration file format is as follows:

```
*x y z c*
```

where x , y , z , denote coordinate of the pixel. c is the value of the concentration.

Example:

```
0 0 0 1.2

0 0 1 1.4

...
```

The initial concentration can also be input from the Python script (typically in the start function of the steppable) but often it is more convenient to type one line of the CC3DML script than few lines in Python.

7.4 Boundary Conditions

All standard solvers (`Flexible`, `Fast`, and `ReactionDiffusion`) by default use the same boundary conditions as the GGH simulation (and those are specified in the Potts section of the CC3DML script). Users can, however, override those defaults and use customized boundary conditions for each field individually. Currently CompuCell3D supports the following boundary conditions for the diffusing fields: periodic, constant value (Dirichlet) and constant derivative (von Neumann). To specify custom boundary condition we include <BoundaryCondition> section inside <DiffusionField> tags.

The <BoundaryCondition> section describes boundary conditions along particular axes. For example:

```
<Plane Axis="X">
    <ConstantValue PlanePosition="Min" Value="10.0"/>
    <ConstantValue PlanePosition="Max"  Value="10.0"/>
</Plane>
```

specifies boundary conditions along the x axis. They are Dirichlet-type boundary conditions. `PlanePosition='Min"` denotes plane parallel to `yz` plane passing through `x=0`. Similarly `PlanePosition="Min"` denotes plane parallel to `yz` plane passing through `x=fieldDimX-1` where `fieldDimX` is x dimension of the lattice.

By analogy we specify constant derivative boundary conditions:

```
<Plane Axis="Y">
    <ConstantDerivative PlanePosition="Min" Value="10.0"/>
    <ConstantDerivative PlanePosition="Max" Value="10.0"/>
</Plane>
```

We can also mix types of boundary conditions along single axis:

```
<Plane Axis="Y">
    <ConstantDerivative PlanePosition="Min" Value="10.0"/>
    <ConstantValue PlanePosition="Max" Value="0.0"/>
</Plane>
```

Here in the `xz` plane at `y=0` we have von Neumann boundary conditions but at `y=fieldFimY-1` we have dirichlet boundary condition.

To specify periodic boundary conditions along, say, x axis we use the following syntax:

```
<Plane Axis="X">
    <Periodic/>
</Plane>
```

Notice, that <Periodic> boundary condition specification applies to both "ends" of the axis *i.e.* we cannot have periodic boundary conditions at `x=0` and constant derivative at `x=fieldDimX-1`.

7.5 DiffusionSolverFE

DiffusionSolverFE is new solver in 3.6.2 and is intended to fully replace `FlexibleDiffusionSolverFE`. It eliminates several limitations and inconveniences of `FlexibleDiffusionSolverFE` and provides new features such as GPU implementation or cell type dependent diffusion/decay coefficients. In addition it also eliminates the need to rescale diffusion/decay/secretion constants. It checks stability condition of the PDE and then rescales appropriately all coefficients and computes how many extra times per MCS the solver has to be called. It makes those extra calls automatically.

> **Warning:** One of the key differences between `FlexibleDiffusionSolverFE` and `DiffusionSolverFE` is the way in which secretion is treated. In `FlexibleDiffusionSolverFE` all secretion amount is done once followed by possibly multiple diffusion calls to diffusion (to avoid numerical instabilities). In `DiffusionSolverFE` the default mode of operation is such that multiple secretion and diffusion calls are interleaved. This means that instead of secreting full amount for a given MCS and diffusing it, the `DiffusionSolverFE` secretes substance gradually so that there is equal amount of secretion before each call of the diffusion. One can change this behavior by adding `<DoNotScaleSecretion/>` to definition of the diffusion solver e.g.
>
> With such definition the `DiffusionSolverFE` will behave like `FlexibleDiffusionSolverFE` as far as computation.

> **Note:** `DiffusionSolverFE` autoscales diffusion discretization depending on the lattice so that `<AutoscaleDiffusion/>` we used in `FlexibleDiffusionSolverFE` is unnecessary. This may result in slow performance so users have to be aware that those extra calls to the solver may be the cause.

Typical syntax for the `DiffusionSolverFE` may look like example below:

```
<Steppable Type="DiffusionSolverFE">
    <DiffusionField Name="ATTR">
        <DiffusionData>
            <FieldName>ATTR</FieldName>
            <GlobalDiffusionConstant>0.1</GlobalDiffusionConstant>
            <GlobalDecayConstant>5e-05</GlobalDecayConstant>
            <DiffusionCoefficient CellType="Red">0.0</DiffusionCoefficient>
        </DiffusionData>
        <SecretionData>
            <Secretion Type="Bacterium">100</Secretion>
        </SecretionData>
        <BoundaryConditions>
            <Plane Axis="X">
                <Periodic/>
            </Plane>
            <Plane Axis="Y">
                <Periodic/>
            </Plane>
            </BoundaryConditions>
    </DiffusionField>
</Steppable>
```

The syntax resembles the syntax for `FlexibleDiffusionSolverFE`. We specify global diffusion constant by using `<GlobalDiffusionConstant>` tag. This specifies diffusion coefficient which applies to entire region of the simulation. We can override this specification for regions occupied by certain cell types by using the following syntax:

```
<DiffusionCoefficient CellType="Red">0.0</DiffusionCoefficient>
```

Similar principles apply to decay constant and we use `<GlobalDecayConstant>` tag to specify global decay coefficient and

```
<DecayCoefficient CellType="Red">0.0</DecayCoefficient>
```

to override global definition for regions occupied by Red cells.

We do not support `<DeltaX>`, `<DeltaT>` or `<ExtraTimesPerMCS>` tags.

Note: `DiffusionSolverFE` autoscales diffusion discretization depending on the lattice so that `<AutoscaleDiffusion/>` we used in `FlexibleDiffusionSolverFE` is unnecessary.

7.5.1 Running DiffusionSolver on GPU

To run `DiffusionSolverFE` on GPU all we have to do (besides having OpenCL compatible GPU and correct drives installed) to replace first line of solver specification:

```
<Steppable Type="DiffusionSolverFE">
```

with

```
<Steppable Type="DiffusionSolverFE_OpenCL">
```

Note: Depending on your computer hardware you may or may not be able to take advantage of GPU capabilities.

7.6 AdvectionDiffusionSolver.

Note: This is an experimental module and was not fully curated.

`AdvectionDiffusionSolver` solves advection diffusion equation on a cell field as opposed to grid. Of course, the inaccuracies are bigger than in the case of PDE being solved on the grid but on the other hand solving the PDE on a cell field means that we associate concentration with a given cell (not just with a lattice point). This means that as cells move so does the concentration. In other words we get advection for free. The mathematical treatment of this kind of approximation was described in Phys. Rev. E 72, 041909 (2005) paper by D.Dan et al.

The equation solved by this steppable is of the type:

$$\frac{\partial c}{\partial t} = D\nabla^2 c + kc + \vec{\nu} \cdot \vec{\nabla} c + \text{secretion} \tag{7.5}$$

where c denotes concentration , D is diffusion constant, k decay constant, $\vec{\nu}$ is velocity field.

In addition to just solving advection-diffusion equation this module allows users to specify secretion rates of the cells as well as different secretion modes. The syntax for this module follows same pattern as `FlexibleDiffusionSolverFE`.

Example syntax:

```
<Steppable Type="AdvectionDiffusionSolverFE">
    <DiffusionField Name="FGF">
        <DiffusionData>
            <FieldName>FGF</FieldName>
            <DiffusionConstant>0.05</DiffusionConstant>
            <DecayConstant>0.003</DecayConstant>
            <ConcentrationFileName>flowFieldConcentration2D.txt</
→ConcentrationFileName>
            <DoNotDiffuseTo>Wall</DoNotDiffuseTo>
        </DiffusionData>
```

(continues on next page)

```
        <SecretionData>
            <Secretion Type="Fluid">0.5</Secretion>
            <SecretionOnContact Type="Fluid"
            <SecreteOnContactWith="Wall">0.3</SecretionOnContact>
        </SecretionData>
    </DiffusionField>
</Steppable>
```

7.7 FastDiffusionSolver2D

Note: The current implementation of `DiffusionSolverFE` may actually be faster than this legacy module. So before commiting to it please verify the performance of teh `DiffusionSolverFE` vs `FastDiffusionSolverFE`

`FastDiffusionSolver2DFE` module is a simplified version of the `FlexibleDiffusionSolverFE` steppable. It runs several times faster that flexible solver but lacks some of its features. Typical syntax is shown below:

```
<Steppable Type="FastDiffusionSolver2DFE">
   <DiffusionField Name="FGF">
     <DiffusionData>
       <UseBoxWatcher/>
       <FieldName>FGF</FieldName>
       <DiffusionConstant>0.010</DiffusionConstant>
       <DecayConstant>0.003</DecayConstant>
     <ExtraTimesPerMCS>2</ExtraTimesPerMCS>
       <DoNotDecayIn>Wall</DoNotDecay>
       <ConcentrationFileName>Demos/diffusion/diffusion_2D_fast_box.pulse.txt
       </ConcentrationFileName>
     </DiffusionData>
   </DiffusionField>
 </Steppable>
```

In particular, for fast solver you cannot specify cells into which diffusion is prohibited. However, you may specify cell types where diffusant decay is prohibited

For explanation on how `<ExtraTimesPerMCS>` tag works works see section on `FlexibleDiffusionSolverFE`.

7.8 KernelDiffusionSolver

This diffusion solver has the advantage over previous solvers that it can handle large diffusion constants. It is also stable. However, it does not accept options like `<DoNotDiffuseTo>` or `<DoNotDecayIn>`. It also requires periodic boundary conditions.

Simply put KernelDiffusionSolver solves diffusion equation:

$$\frac{\partial c}{\partial t} = D\nabla^2 c + kc + \text{secretion} \tag{7.6}$$

with fixed, periodic boundary conditions on the edges of the lattice. This is different from `FlexibleDiffusionSolverFE` where the boundary conditions evolve. You also need to choose a proper `Kernel range` (K) according to the value of diffusion constant. Usually when $K^2 e^{-K^2/4D}$ is small (this is the main part of the integrand), the approximation converges to the exact value.

The syntax for this solver is as follows:

```
<Steppable Type="KernelDiffusionSolver">
  <DiffusionField Name="FGF">
    <Kernel>4</Kernel>
    <DiffusionData>
      <FieldName>FGF</FieldName>
      <DiffusionConstant>1.0</DiffusionConstant>
      <DecayConstant>0.000</DecayConstant>
    <ConcentrationFileName>
      Demos/diffusion/diffusion_2D.pulse.txt
      </ConcentrationFileName>
    </DiffusionData>
  </DiffusionField>
</Steppable>
```

Inside `<DiffusionField>` tag one may also use option `<CoarseGrainFactor>`. For example:

```
<Steppable Type="KernelDiffusionSolver">
  <DiffusionField Name="FGF">
    <Kernel>4</Kernel>
    <CoarseGrainFactor>2</CoarseGrainFactor>
    <DiffusionData>
      <FieldName>FGF</FieldName>
      <DiffusionConstant>1.0</DiffusionConstant>
      <DecayConstant>0.000</DecayConstant>
      <ConcentrationFileName>
      Demos/diffusion/diffusion_2D.pulse.txt
      </ConcentrationFileName>
    </DiffusionData>
  </DiffusionField>
</Steppable>
```

7.9 ReactionDiffusionSolver

The reaction diffusion solver solves the following system of N reaction diffusion equations:

$$\frac{\partial c_1}{\partial t} = D\nabla^2 c_1 + kc_1 + \text{secretion} + f_1(c_1, c_2, ..., c_N, W)$$
$$\frac{\partial c_2}{\partial t} = D\nabla^2 c_2 + kc_2 + \text{secretion} + f_2(c_1, c_2, ..., c_N, W)$$
$$...$$
$$\frac{\partial c_N}{\partial t} = D\nabla^2 c_N + kC_N + \text{secretion} + f_N(c_1, c_2, ..., c_N, W)$$

where `W` denotes cell type

Let's consider a simple example of such system:

$$\frac{\partial F}{\partial t} = 0.1\nabla^2 F - 0.1H$$
$$\frac{\partial H}{\partial t} = 0.0\nabla^2 H + 0.1F$$

It can be coded as follows:

```
<Steppable Type="ReactionDiffusionSolverFE">
  <AutoscaleDiffusion/>
  <DiffusionField Name="F">
    <DiffusionData>
      <FieldName>F</FieldName>
      <DiffusionConstant>0.010</DiffusionConstant>
      <ConcentrationFileName>
      Demos/diffusion/diffusion_2D.pulse.txt
      </ConcentrationFileName>
      <AdditionalTerm>-0.01*H</AdditionalTerm>
    </DiffusionData>
  </DiffusionField>

  <DiffusionField Name="H">
    <DiffusionData>
      <FieldName>H</FieldName>
      <DiffusionConstant>0.0</DiffusionConstant>
      <AdditionalTerm>0.01*F</AdditionalTerm>
    </DiffusionData>
  </DiffusionField>
</Steppable>
```

Notice how we implement functions f from the general system of reaction diffusion equations. We simply use `<AdditionalTerm>` tag and there we type arithmetic expression involving field names (tags `<FieldName>`). In addition to this we may include in those expression word `CellType`. For example:

```
<AdditionalTerm>0.01*F*CellType</AdditionalTerm>
```

This means that function f will depend also on `CellType`. `CellType` holds the value of the type of the cell at particular location - x, y, z - of the lattice. The inclusion of the cell type might be useful if you want to use additional terms which may change depending of the cell type. Then all you have to do is to either use if statements inside `<AdditionalTerm>` or form equivalent mathematical expression using functions allowed by `muParser`: http://muparser.sourceforge.net/mup_features.html#idDef2

For example, let's assume that additional term for second equation is the following: $f_F = \begin{cases} 0.1F \\ \text{if CellType=1} \\ 0.51F \\ \text{otherwise} \end{cases}$ $In such a case additional term would be coded as follows:$

```
<AdditionalTerm>CellType==1 ? 0.01*F : 0.15*F</AdditionalTerm>
```

Notice that we have used here, so called ternary operator which might be familiar to you from other programing languages such as C or C++ and is equivalent to`` if-then-els``e statement

The syntax of the ternary (aka `if-then-else` statement) is as follows:

```
condition ? expression if condition is true : expression if condition false
```

> **Warning:** **Important:** If change the above expression to
>
> we will get an XML parsing error. Why? This i because XML parser will think that `<1` is the beginning of the new XML element. To fix this you could use two approaches:
>
> 1.Present your expression as `CDATA`

```
<AdditionalTerm>
    <![CDATA[
    CellType<1 ? 0.01*F : 0.15*F
    ]]>
</AdditionalTerm>
```

In this case XML parser will correctly interpret the expression enclosed between `<![CDATA[` and `]]>`.

2. Replace XML using equivalent Python syntax - see (http://pythonscriptingmanual.readthedocs.io/en/latest/replacing_cc3dml_with_equivalent_python_syntax.html) in which case you would code the above XML element as the following Python statement:

```
DiffusionDataElmnt\_2.ElementCC3D('AdditionalTerm', {}, 'CellType<1 ? 0.01*F : 0.
↪15*F')
```

The moral from this story is that if like to use muParser in the XML file make sure to use this general syntax:

```
<AdditionalTerm>
    <![CDATA[
        YOUR EXPRESSION
    ]]>
</AdditionalTerm>
```

One thing to remember is that computing time of the additional term depends on the level of complexity of this term. Thus, you might get some performance degradation for very complex expressions coded in muParser

Similarly as in the case of `FlexibleDiffusionSolverFE` we may use `<AutoscaleDiffusion>` tag tells CC3D to automatically rescale diffusion constant. See section `FlexibleDiffusionSolver` or the `Appendix` for more information.

7.10 Steady State diffusion solver

Often in the multi-scale simulations we have to deal with chemicals which have drastically different diffusion constants. For slow diffusion fields we can use standard explicit solvers (*e.g.* `FlexibleDiffusionSolverFE`) but once the diffusion constant becomes large the number of extra calls to explicit solvers becomes so large that solving diffusion equation using Forward-Euler based solvers is simply impractical. In situations where the diffusion constant is so large that the solution of the diffusion equation is not that much different from the asymptotic solution (i.e. at $t = \infty$) it is often more convenient to use `SteadyStateDiffusion` solver which solves Helmholtz equation:

$$\nabla^2 c - kc = F \tag{7.7}$$

where F is a source function of the coordinates - it is an input to the equation, k is decay constant and c is the concentration. The F function in CC3D is either given implicitly by specifying cellular secretion or explicitly by specifying concentration c before solving Helmholtz equation.

The CC3D stead state diffusion solvers are stable and allow solutions for large values of diffusion constants. The example syntax for the steady-state solver is shown below:

```
<Steppable Type="SteadyStateDiffusionSolver2D">
    <DiffusionField Name="INIT">
        <DiffusionData>
            <FieldName>INIT</FieldName>
            <DiffusionConstant>1.0</DiffusionConstant>
            <DecayConstant>0.01</DecayConstant>
        </DiffusionData>
```

(continues on next page)

```
        <SecretionData>
            <Secretion Type="Body1">1.0</Secretion>
        </SecretionData>


        <BoundaryConditions>


        <Plane Axis="X">
            <ConstantValue PlanePosition="Min" Value="10.0"/>
            <ConstantValue PlanePosition="Max"  Value="5.0"/>
        </Plane>


        <Plane Axis="Y">
            <ConstantDerivaive PlanePosition="Min" Value="0.0"/>
            <ConstantDerivaive PlanePosition="Max"  Value="0.0"/>
        </Plane>


        </BoundaryConditions>


    </DiffusionField>

</Steppable>
```

The syntax is is similar (actually, almost identical) to the syntax of the `FlexibleDiffusionSolverFE`. The only difference is that while `FlexibleDiffusionSolverFE` works in in both 2D and 3D users need to specify the dimensionality of the steady state solver. We use

```
<Steppable Type="SteadyStateDiffusionSolver2D">
```

for 2D simulations when all the cells lie in the xy plane and

```
<Steppable Type="SteadyStateDiffusionSolver">
```

for simulations in 3D.

Note: We can use Python to control secretion in the steady state solvers but it requires a little bit of low level coding. Implementing secretion in steady state diffusion solver is different from "regular" Forward Euler solvers. Steady state solver takes secretion rate that is specified at $t=0$ and returns the solution at $t=\infty$. For a large diffusion constants we approximate solution to the PDE during one MCS by using solution at`` t=$\infty$``. However, this means that if at each MCS secretion changes we have to do three things 1) zero entire field, 2) set secretion rate 3) solve steady state solver. The reason we need to zero entire field is because any value left in the field at `mcs=N` is interpreted by the solver as a secretion constant at this location at `mcs=N+1`. **Moreover, the the secretion constant needs to have negative value if we want to secrete positive amount of substance - this weird requirements comes from the fact that we re using 3:sup:'rd' party solver which inverts signs of the secretion constants.**

An example below demonstrates how we control secretion of the steady state in Python. First we need to include tag `<ManageSecretionInPython/>` in the XML definition of the solver:

```
<Steppable Type="SteadyStateDiffusionSolver2D">
    <DiffusionField>
        <ManageSecretionInPython/>
        <DiffusionData>
            <FieldName>FGF</FieldName>
            <DiffusionConstant>1.00</DiffusionConstant>
            <DecayConstant>0.00001</DecayConstant>
```

```
            </DiffusionData>
        </DiffusionField>
</Steppable>
```

In Python the code to control the secretion involves iteration over every pixel and adjusting concentration (which as we mentioned will be interpreted by the solver as a secretion constant) and we have to make sure that we inherit from `SecretionBasePy` not `SteppableBasePy` to ensure proper ordering of calls to Python module and the C++ diffusion solver.

Note: Make sure you inherit from `SecretionBasePy` when you try to manage secretion in the steady state solver using Python. This will ensure proper ordering of calls to steppable and to C++ solver code.

Note: Once you use `<ManageSecretionInPython/>` tag in the XML all secretion tags in the `SecretionData` will be ignored. In other words, for this solver you cannot mix secretion specification in Python and secretion specification in the XML.

```python
def __init__(self, _simulator, _frequency=1):
    SecretionBasePy.__init__(self, _simulator, _frequency)


def start(self):
    self.field = CompuCell.getConcentrationField \
        (self.simulator, "FGF")

    secrConst = 10
    for x, y, z in self.everyPixel(1, 1, 1):
        cell = self.cellField[x, y, z]
        if cell and cell.type == 1:
            self.field[x, y, z] = -secrConst
        else:
            self.field[x, y, z] = 0.0


def step(self, mcs):
    secrConst = mcs
    for x, y, z in self.everyPixel(1, 1, 1):
        cell = self.cellField[x, y, z]
        if cell and cell.type == 1:
            self.field[x, y, z] = -secrConst
        else:
            self.field[x, y, z] = 0.0
```

Warning: Notice that all the pixels that do not secrete **have to be 0.0** as mentioned above. **If you don't initialize field values in the non-secreting pixels to 0.0 you will get wrong results**. The above code, with comments, is available in our Demo suite (`Demos/SteppableDemos/SecretionSteadyState` or `Demos/SteppableDemos/SteadyStateDiffusionSolver`).

7.11 Fluctuation Compensator Solver Add-On

Fluctuation Compensator is an optional PDE solver add-on introduced in v4.1.2 to account for Metropolis surface fluctuations in field solutions in both a feasible and sensible way. The algorithm is based on that which is described by Marée et. al[1] and imposes total mass conservation in all cellular domains and the medium over the spin flips of a Monte Carlo step. The algorithm does not impose advection by domain deformation, but rather homogeneously applies a correction factor in each subdomain to all field solutions of a solver such that the total amount of each simulated species in each subdomain is unchanged over all spin flips.

The Fluctuation Compensator algorithm consists of the following three rules.

1. **Rule 1 (mass conservation)**: In each subdomain, the total amount of each species is unchanged over an arbitrary number of spin flips.

2. **Rule 2 (uniform correction)**: Corrections applied to values in each subdomain so to impose mass conservation are uniformly applied.

3. **Rule 3 (Neumann condition)**: The amount of species in the copying site of a spin flip are exactly copied to the site of the flip.

Note: Fluctuation Compensator is supported in DiffusionSolverFE and ReactionDiffusionSolverFE.

Exactly one Fluctuation Compensator can be attached to each supported solver instance. An attached Fluctuation Compensator performs corrections on *all* fields of the solver to which it is attached. A Fluctuation Compensator can be attached to a solver in CC3DML using the tag `<FluctuationCompensator/>` at the level of field specification. For example, to attach a Fluctuation Compensator to a simple use-case with DiffusionSolverFE might look like the following.

```
<Steppable Type="DiffusionSolverFE">
    <FluctuationCompensator/>
    <DiffusionField Name="ATTR"/>
</Steppable>
```

Demos showing basic usage and comparison are available in `Demos/SteppableDemos/FluctuationCompensator`.

Note: PDE solution field values can be modified outside of the solver routines without invalidating the correction factors of Fluctuation Compensators *so long as* they are notified that field values have been modified. The CompuCell3D library has a convenience method to do exactly this: `updateFluctuationCompensators`. Call this method after modifying field values and before the next PDE solution step to refresh Fluctuation Compensators according to your changes.

---

[1] Marée, Athanasius FM, Verônica A. Grieneisen, and Leah Edelstein-Keshet. "How cells integrate complex stimuli: the effect of feedback from phosphoinositides and cell shape on cell polarization and motility." PLoS Computational Biology 8.3 (2012).

Appendix

This section covers some of the computational algorithms we use in CC3D

- inertia_tensor
- calculating_elongation_term
- forward_euler
- com_with_periodic_bc
- dividing_clusters
- command_line_options
- cc3d_project_files

8.1 Calculating Inertia tensor In CC3D

For each cell the inertia tensor is defined as follows:

$$I = \begin{bmatrix} \sum_i y_i^2 + z_i^2 & -\sum_i x_i y_i & -\sum_i x_i z_i \\ -\sum_i x_i y_i & \sum_i x_i^2 + z_i^2 & -\sum_i y_i z_i \\ -\sum_i x_i z_i & -\sum_i y_i z_i & \sum_i x_i^2 + y_i^2 \end{bmatrix} \quad (8.1)$$

where index i denotes `i-th` pixel of a given cell and x_i, y_i and z_i are coordinates of that pixel in a given coordinate frame.
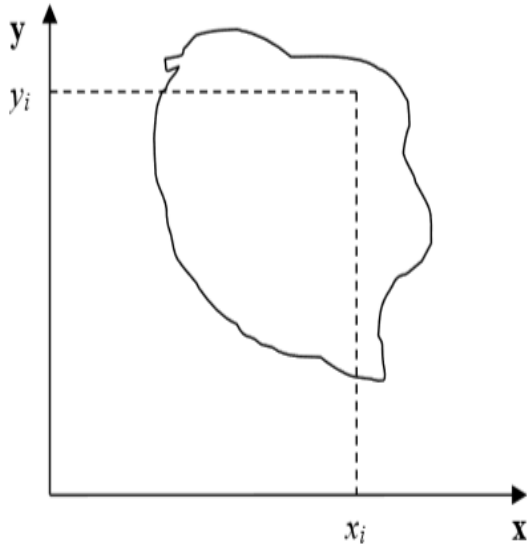
Figure 3: Cell and coordinate system passing through center of mass of a cell. Notice that as cell changes shape the position of center of mass moves.
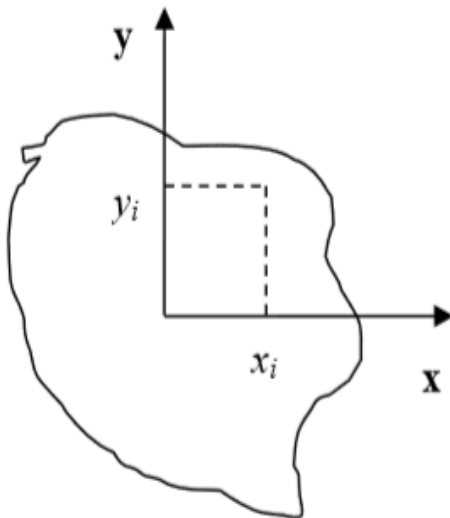

Figure 4: Cell and its coordinate frame in which we calculate inertia tensor

In Figure 4 we show one possible coordinate frame in which one can calculate inertia tensor. If the coordinate frame is fixed calculating components of inertia tensor for cell gaining or losing one pixel is quite easy. We will be adding and subtracting terms like $y_i^2 + z_i^2$ or $x_i z_i$.

However, in CompuCell3D we are mostly interested in knowing tensor of inertia of a cell with respect to `xyz` coordinate frame with origin at the center of mass (*COM*) of a given cell as shown in Figure 3. Now, to calculate such tensor we cannot simply add or subtract terms $y_i^2 + z_i^2$ or $x_i z_i$ to account for lost or gained pixel. If a cell gains or loses a pixel its `COM` coordinates change. If so then all the x_i,:math:`y_i`, z_i coordinates that appear in the inertia tensor expression will have different values. Thus, for each change in cell shape (gain or loss of pixel) we would have to recalculate inertia tensor from scratch. This would be quite time consuming and would require us to keep track of all the pixels belonging to a given cell. It turns out, however, that there is a better way of keeping track of inertia tensor for cells. We will be using parallel axis theorem to do the calculations. Parallel axis theorem states that if I_{COM} is a moment of inertia with respect to axis passing through center of mass then we can calculate moment of inertia with

respect to any parallel axis to the one passing through the COM by using the following formula:

$$I_{x'x'} = I_{xx} + Md^2 \tag{8.2}$$

where I_{xx} denotes moment of inertia with respect to x axis passing through center of mass, $I_{x'x'}$ is a moment of inertia with respect to some other axis parallel to the x , d is the distance between the axes two and M is mass of the cell.
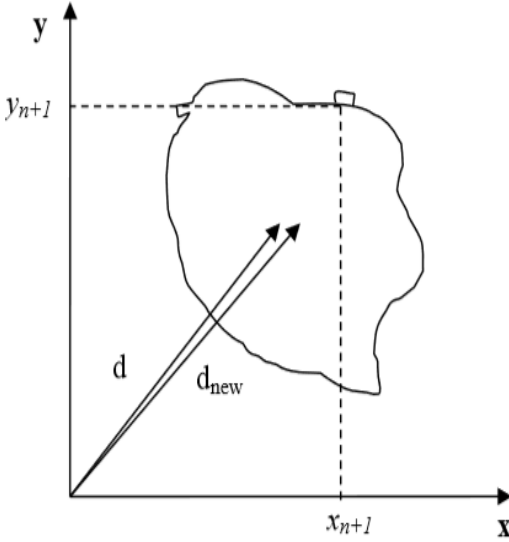
Let us now draw a picture of a cell gaining one pixel:

Figure 5: Cell gaining one pixel.d denotes a distance from origin of a fixed frame of reference to a center of mass of a cell before cell gains new pixel. d_{new} denotes same distance but after cell gains new pixel

Now using parallel axis theorem we can write expression for the moment of inertia after cell gains one pixel the following that:

$$I_{xx}^{new} = I_{x'x'}^{new} - (V + 1)d_{new}^2 \tag{8.3}$$

where, as before, I_{xx}^{new} denotes moment of inertia of a cell with new pixel with respect to x axis passing through center of mass, $I_{x'x'}^{new}$ is a moment of inertia with respect to axis parallel to the x axis passing through center of mass, d_{new} is the distance between the axes and $V + 1$ is volume of the cell **after** it gained one pixel. Now let us rewrite above equation by adding ad subtracting Vd^2 term:

$$I_{xx}^{new} = I_{x'x'}^{old} + y_{n+1}^2 + z_{n+1}^2 - Vd^2 + Vd^2(V + 1)d_{new}^2 \tag{8.4}$$

$$= I_{x'x'}^{old} - Vd^2 + y_{n+1}^2 + z_{n+1}^2 + Vd^2(V + 1)d_{new}^2 \tag{8.5}$$

$$I_{xx}^{old} - Vd^2 + y_{n+1}^2 + z_{n+1}^2 + Vd^2(V + 1)d_{new}^2 \tag{8.6}$$

Therefore we have found an expression for moment of inertia passing through the center of mass of the cell with additional pixel. Note that this expression involves moment of inertia but for the old cell (*i.e.* the original cell, not the one with extra pixel). When we add new pixel we know its coordinates and we can also easily calculate d_{new} . Thus, when we need to calculate the moment of inertia for new cell instead of performing summation as given in the definition of the inertia tensor we can use much simpler expression.

This was diagonal term of the inertia tensor. What about off-diagonal terms? Let us write explicitly expression for I_{xy}

:

$$I_{xy} = -\sum_i^N (x_i - x_{com})(y_i - y_{com}) = -\sum_i^N x_i y_i + x_{COM}\sum_i^N y_i + y_{COM}\sum_i^N x_i - x_{COM}y_{COM}\sum_i^N \quad (8.7)$$

$$= -\sum_i^N x_i y_i + x_{COM}V y_{COM} + y_{COM}V x_{COM} - x_{COM}y_{COM}V \quad (8.8)$$

$$= -\sum_i^N x_i y_i + V x_{COM}y_{COM} \quad (8.9)$$

where x_{COM}, y_{COM} denote x and y center of mass positions of the cell, V denotes cell volume. In the above formula we have used the fact that:

$$x_{COM} = \frac{\sum_i x_i}{V} \implies \sum_i x_i = x_{COM}V \quad (8.10)$$

and similarly for the y coordinate.

Now, for the new cell with additional pixel we have the following relation:

$$I_{xy}^{new} = -\sum_i^{N+1} x_i y_i + (V+1)x_{COM}^{new}y_{COM}^{new} \quad (8.11)$$

$$= -\sum_i^N x_i y_i + V x_{COM}y_{COM} - x_{COM}V y_{COM} + (V+1)x_{COM}^{new}y_{COM}^{new} - x_{N+1}y_{n+1} \quad (8.12)$$

$$= I_{xy}^{old} - V x_{COM}y_{COM} + (V+1)x_{COM}^{new}y_{COM}^{new} - x_{N+1}y_{n+1} \quad (8.13)$$

where we have added and subtracted $V x_{COM}y_{COM}$ to be able to form $I_{xy}^{old} - \sum_i^N x_i y_i + V x_{COM}y_{COM}$ on the right hand side of the expression for I_{xy}^{new}. As it was the case for diagonal element, calculating off-diagonal of the inertia tensor involves ::math'I_{xy}^{old}' and positions of center of mass of the cell before and after gaining new pixel. All those quantities are either known *a priori* (::math'I_{xy}^{old}') or can be easily calculated (center of mass position after gaining one pixel).

Therefore, we have shown how we can calculate tensor of inertia for a given cell with respect to a coordinate frame with origin at cell's center of mass, without evaluating full sums. Such "local" calculations greatly speed up simulations

8.2 Calculating shape constraint of a cell – elongation term

The shape of single cell immersed in medium and not subject to too drastic surface or surface constraints will be spherical (circular in 2D). However in certain situation we may want to use cells which are elongated along one of their body axes. To facilitate this we can place constraint on principal lengths of cell. In 2D it is sufficient to constrain one of the principal lengths of cell how ever in 3D we need to constrain 2 out of 3 principal lengths. Our first task is to diagonalize inertia tensor (i.e. find a coordinate frame transformation which brings inertia tensor to a diagonal form)

8.2.1 Diagonalizing inertia tensor

We will consider here more difficult 3D case. The 2D case is described in detail in M.Zajac, G.L.jones, J,A,Glazier "*Simulating convergent extension by way of anisotropic differential adhesion*" Journal of Theoretical Biology **222** (2003) 247–259.

In order to diagonalize inertia tensor we need to solve eigenvalue equation:

$$det(I - \lambda) = 0 \quad (8.14)$$

or in full form

$$
0 = \begin{vmatrix} \sum_i y_i^2 + z_i^2 - \lambda & -\sum_i x_i y_i & -\sum_i x_i z_i \\ -\sum_i x_i y_i & \sum_i x_i^2 + z_i^2 - \lambda & -\sum_i y_i z_i \\ -\sum_i x_i z_i & -\sum_i y_i z_i & \sum_i x_i^2 + y_i^2 - \lambda \end{vmatrix} = \begin{vmatrix} I_{xx} - \lambda & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} - \lambda & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} - \lambda \end{vmatrix} \quad (8.15)
$$

The eigenvalue equation will be in the form of 3rd order polynomial. The roots of it are guaranteed to be real. The polynomial itself can be found either by explicit derivation, using symbolic calculation or simply in Wikipedia (http://en.wikipedia.org/wiki/Eigenvalue_algorithm)

$$
\det \begin{bmatrix} a - \lambda & b & c \\ d & e - \lambda & f \\ g & h & i - \lambda \end{bmatrix} = -\lambda^3 + \lambda^2(a+e+i) + \lambda(db+gc+fh-ae-ai-ei) + (aei - afh - db
$$

so in our case the eigenvalue equation takes the form:

$$
-L^2 + L^2(I_{xx} + I_{yy} + I_{zz}) + L(I_{xy}^2 + I_{yz}^2 + I_{xz}^2 - I_{xx}I_{yy} - I_{yy}I_{zz} - I_{xx}I_{zz}) \quad (8.16)
$$

$$
+ I_{xx}I_{yy}I_{zz} - I_{xx}I_{yz}^2 - I_{yy}I_{zz}^2 I_{zz}I_{xy}^2 + 2I_{xy}I_{yz}I_{zx} \quad (8.17)
$$

This equation can be solved analytically, again we may use Wikipedia (http://en.wikipedia.org/wiki/Cubic_function)

Now, the eigenvalues found that way are principal moments of inertia of a cell. That is they are components of inertia tensor in a coordinate frame rotated in such a way that off-diagonal elements of inertia tensor are 0: I =

$$
\begin{vmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{vmatrix} \quad (8.18)
$$

In our cell shape constraint we will want to obtain ellipsoidal cells. Therefore the target tensor of inertia for the cell should be tensor if inertia for ellipsoid: I =

$$
\begin{vmatrix} \frac{1}{5}(b^2 + c^2) & 0 & 0 \\ 0 & \frac{1}{5}(a^2 + c^2) & 0 \\ 0 & 0 & \frac{1}{5}(a^2 + b^2) \end{vmatrix} \quad (8.19)
$$

where a, b, c are parameters describing the surface of an ellipsoid:

$$
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (8.20)
$$

In other words a, b, c are half lengths of principal axes (they are analogues of circle's radius).

Now we can determine semi axes lengths in terms of principal moments of inertia by inverting the following set of equations:

$$
I_{xx} = \frac{1}{5}(b^2 + c^2) \quad (8.21)
$$

$$
I_{yy} = \frac{1}{5}(a^2 + c^2) \quad (8.22)
$$

$$
I_{zz} = \frac{1}{5}(a^2 + b^2) \quad (8.23)
$$

Once we have calculated semiaxes lengths in terms of moments of inertia we can plug –in actual numbers for moment of inertia (the ones for actual cell) and obtain lengths of semiexes. Next we apply quadratic constraint on largest (semimajor) and smallest (seminimor axes). This is what elongation plugin does.

8.2. Calculating shape constraint of a cell – elongation term

8.3 Forward Euler method for solving PDE's in CompuCell3D.

Note: We present more complete derivations of explicit finite difference scheme for diffusion solver in "Introduction to Hexagonal Lattices in CompuCell3D" (http://www.compucell3d.org/BinDoc/cc3d_binaries/Manuals/HexagonalLattice.pdf).

In CompuCell3D most of the solvers uses explicit schemes (Forward Euler method) to obtain PDE solutions. Thus for the diffusion equation we have:

$$\frac{\partial c}{\partial t} = \frac{\partial^2 c}{\partial^2 x} + \frac{\partial^2 c}{\partial^2 y} + \frac{\partial^2 c}{\partial^2 z} \tag{8.24}$$

In a discretetized form we may write:

$$\frac{c(x, t + \delta t) - c(x, t)}{\delta t} = \tag{8.25}$$

$$\frac{c(x + \delta x, t) - 2c(x, t) + c(x - \delta x, t)}{\delta x^2} \tag{8.26}$$

$$+ \frac{c(y + \delta y, t) - 2c(y, t) + c(y - \delta y, t)}{\delta y^2} \tag{8.27}$$

$$+ \frac{c(z + \delta z, t) - 2c(z, t) + c(z - \delta z, t)}{\delta z^2} \tag{8.28}$$

where to save space we used shorthand notation:

$$c(x + \Delta x, y, z, t)) \equiv c(x + \Delta x, , t)) \tag{8.29}$$

$$c(x, y, z, t) \equiv c(x, t) \tag{8.30}$$

and similarly for other coordinates.

After rearranging terms we get the following expression:

$$c(x, t + \delta t) = \left[\frac{\delta t}{\delta x^2} \sum_{i=neighbors} (c(i, t) - c(x, t)) \right] - c(x, t) \tag{8.31}$$

where the sum over index i goes over neighbors of point (x, y, z) and the neighbors will have the following concentrations: $c(x + \delta x, t)$, $c(y + \delta y, t)$, $c(z + \delta z, t)$.

8.4 Calculating center of mass when using periodic boundary conditions.

When you are running calculation with periodic boundary condition you may end up with situation like in the figure below:
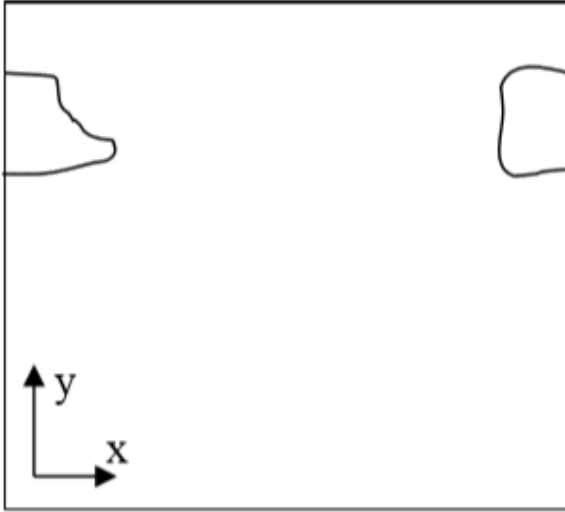
Figure 6 A connected cell in the lattice edge area – periodic boundary conditions are applied

Clearly, what happens is that simply connected cell is wrapped around the lattice edge so part of it is in the region of high values of x coordinate and the other is in the region where x coordinates have low values. Consequently, a naïve calculation of center of mass position according to:

$$x_{COM} = \frac{\sum_i x_i}{V} \tag{8.32}$$

or in vector form:

$$\vec{r}_{COM} = \frac{\sum_i \vec{r}_i}{V} \tag{8.33}$$

would result in being somewhere in the middle of the lattice and obviously outside the cell. A better procedure could be as follows:

Before calculating center of mass when new pixel is added or lost we "shift" a cell and new pixel (gained or lost)to the middle of the lattice do calculations "in the middle of the lattice" and shift back. Now if after shifting back it turns out that center of mass of a cell lies outside lattice position it in the center of mass by applygin a shift equal to the length of the lattice and whose direction should be such that the center of mass of the cell ends up inside the lattice (there is only one such shift and it might be be equal to zero vector).

This is how we do it using mathematical formulas:

$$\vec{s} = \vec{r}_{COM} - \vec{c} \tag{8.34}$$

First we define shift vector $\vec{s}$ as a vector difference between vector pointing to center of mass of the cell $\vec{r}_{COM}$ and vector pointing to (approximately) the middle of the lattice $\vec{c}$.

Next we shift cell to the middle of the lattice using :

$$\vec{r'}_{COM} = \vec{r}_{COM} - \vec{s} \tag{8.35}$$

where $\vec{r'}_{COM}$ denotes center of mass position of a cell after shifting but before adding or subtracting a pixel.

Next we take into account the new pixel (either gained or lost) and calculate center of mass position (for the shifted cell):

$$\vec{r'}_{COM}^{new} = \frac{\vec{r'}_{COM}V + \vec{r}_i}{V + 1} \tag{8.36}$$

8.4. Calculating center of mass when using periodic boundary conditions. **75**

Above we have assumed that we are adding one pixel.

Now all that we need to do is to shift back $\vec{r'}_{COM}^{new}$ by same vector $\vec{s}$ that brought cell to (approximately) center of the lattice:

$$\vec{r}_{COM}^{new} = \vec{r'}_{COM}^{new} + \vec{s} \tag{8.37}$$

We are almost done. We still have to check if $\vec{r'}_{COM}^{new}$ is inside the lattice. If this is not the case we need to shift it back to the lattice but now we are allowed to use only a vector $\vec{P}$ whose components are multiples of lattice dimensions (and we can safely restrict to +1 and -1 multiples of the lattice dimensions) . For example we may have:

$$\vec{P} = (x_{max}, -y_{max}, 0) \tag{8.38}$$

where $\vec{x}_{max}$, $\vec{y}_{max}$, $\vec{z}_{max}$ are dimensions of the lattice.

There is no cheating here. In the lattice with periodic boundary conditions you are allowed to shift point coordinates a vector whose components are multiples of lattice dimensions.

All we need to do is to examine new center of mass position and form suitable vector $\vec{P}$.

8.5 Dividing cluster cells

While dividing non-clustered cells is straightforward, doing the same for clustered cells is more challenging. To divide non-cluster cell using directional mitosis algorithm we construct a line or a plane passing through center of mass of a cell and pixels of the cell (we are using PixelTracker plugin with mitosis) on one side of the line/plane end up in child cell and the rest stays in parent cell. The orientation of the line/plane can be either specified by the user or we can use CC3D built-in feature to calculate calculate orientation tion of principal axes and divide either along minor or major axis.

With compartmental cells, things get more complicated because: 1) Compartmental cells are composed of many subcells. 2) There can be different topologies of clusters. Some clusters may look "snake-like" and some might be compactly packed blobs of subcells. The algorithm which we implemented in CC3D works in the following way:

1) We first construct a set of pixels containing every pixel belonging to a cluster cell. You may think of it as of a single "regular" cell.

2) We store volumes of compartments so that we know how big compartments should be after mitosis (they will be half of original volume)

3) We calculate center of mass of entire cluster and calculate vector offsets between center of mass of a cluster and center of mass of particular compartments as on the figure below:
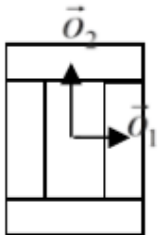

Figure 7.Vectors $\vec{o}_1$ and $\vec{o}_2$ show offsets between center of mass of a cluster and center of mass particular compartments.

4) We pick division line/plane and for parents and child cells with offsets between cluster center of mass (after mitosis) and center of masses of clusters. We do it according to the formula:

$$\vec{p} = \vec{o} - \frac{1}{2}(\vec{o} \cdot \vec{n})\vec{n} \tag{8.39}$$

where $\vec{p}$ denotes offset after mitosis from center of mass of child (parent) clusters, $\vec{o}$ is orientation vector before mitosis (see picture above) and $\vec{n}$ is a normalized vector perpendicular to division line/plane. If we try to divide the cluster along dashed line as on the picture below:
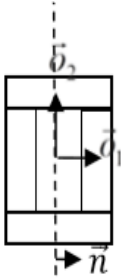

Figure 8. Division of cell along dashed line. Notice the orientation of $\vec{n}$. The offsets after the mitosis for child and parent cell will be $\vec{p}_1 = \frac{1}{2}\vec{o}_1$ and $\vec{p}_2 = \vec{o}_2$ as expected because both parent and child cells will retain their heights but after mitosis will become twice narrower (cell with grey outer
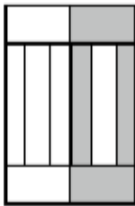
compartments is a parent cell):

Figure 9.Child and parent (the one with grey outer compartments) cells after mitosis.

The formula given above is heuristic. It gives fairly simple way of assigning pixels of child/parent clusters to cellular compartments. It is not perfect but the idea is to get approximate shape of the cell after the mitosis and as simulation runs cell shape will readjust based on constraints such as adhesion of focal point plasticity. Before continuing with the mitosis we check if center of masses of compartments belong to child/parent clusters. If the center of masses are outside their target pixels we abandon mitosis and wait for readjustment of cell shape at which point mitosis algorithm will pass this sanity check. For certain "exotic" shapes of cluster shapes presented mitosis algorithm may not work well or at all. In this case we would have to write specialized mitosis algorithm.

5) We divide clusters and knowing offsets from child/parent cluster center of mass we assign pixels to particular compartments. The assignment is based on the distance of particular pixel to center of masses of clusters. Pixel is assigned to particular compartment if its distance to the center of mass of the compartment is the smallest as compared to distances between centroids of other compartments. If given compartment has reached its target volume and other compartments are underpopulated we would assign pixels to other compartments based on the closest distance criterion. Although this method may result in some deviation from perfect 50-50 division of compartment volume in most cases after few MCS cells will readjust their volume.

Figure 10. CC3D example of compartmental cell division. See also *Demos/CompuCellPythonTutorial/clusterMitosis.*

8.6 Command line options of CompuCell3D

Although most users run CC3D using Player GUI sometimes it is very convenient to run CC3D using command line options. CC3D allows to invoke Player directly from command line which is convenient because if saves several clicks and if you run many simulations this might be quite convenient.

Remark: On Windows we use .bat extension for run scripts and on Linux/OSX it is .sh. Otherwise all the material in this section applies to all the platforms.

8.6.1 CompuCell3D Player Command Line Options

The command line options for running simulation with the player are as follows:

```
compucell3d.bat [options]
```

Options are:

`-i <simulation file>` - users specify `.cc3d` simulation file they want to run.

`-s <screenshotDescriptionFileName>` - name of the file containing description of screenshots to be taken with the simulation. Usually this file is prepared using Player by switching to different views, clickin camera button and saving screenshot description file from the Player File menu.

`-o <customScreenshotDirectoryName>` - allows users to specify where screenshots will be written. Overrides default settings.

`--noOutput` - instructs CC3D not to store any screenshots. Overrides Player settings.

`--exitWhenDone` - instructs CC3D to exit at the end of simulation. Overrides Player settings.

`-h, --help` - prints command line usage on the screen

Example command may look like (windows):

```
compucell3d.bat -i Demos\Models\cellsort\cellsort_2D\cellsort_2d.cc3d --noOutput
```

or on linux:

```
compucell3d.sh -i Demos/Models/cellsort/cellsort_2D/cellsort_2d.cc3d --noOutput
```

and OSX:

```
compucell3d.command -i Demos/Models/cellsort/cellsort_2D/cellsort_2d.cc3d --noOutput
```

8.6.2 Running CompuCell3D in a GUI-Less Mode - Command Line Options.

Sometimes when you want to run CC3D on a cluster you will have to use runScript.bat which allows running CC3D simulations without invoking GUI. However, all the screenshots will be still stored.

Remark: current version of this script does not handle properly relative paths so it has to be run from the installation directory of CC3D i.e. you have to cd into this directory prior to runnit runScript.bat. Another solution is to use full paths.

The output of this script is in the form of vtk files which can be subsequently replayed in the Player (and one can take screenshots then). By default all fields present in the simulation are stored in the vtk file. If users want to remove some of the fields from being stored in the vtk format they have to pass this information in the Python script:

```
CompuCellSetup.doNotOutputField(_fieldName)
```

Storing entire fields (as opposed to storing screenshots) preserves exact snapshots of the simulation and allows result postprocessing. In addition to the vtk files runScript stores lattice description file with `.dml`` extension which users open in the Player (`File->Open Lattice Description Summary File...`) if they want to reply generated vtk files.

The format of the command (windows):

```
runScript.bat [options]
```

linux:

```
runScript.sh [options]
```

OSX:

```
runScript.command [options]
```

The command line options for runScript.bat are as follows:

`-i <simulation file>` - users specify `.cc3d` simulation file they want to run.

`-c <outputFileCoreName>` - allows users to specify core name for the vtk files. The default name for vtk files is `Step`

`-o <customVtkDirectoryName>` - allows users to specify where vtk files and the .dml file will be written. Overrides default settings

`-f <frequency>` or `-outputFrequency=<frequency>` - allows to specify how often vtk files are stored to the disk. Those files tend to be quite large for bigger simulations so storing them every single MCS (default setting) slows down simulation considerably and also uses a lot of disk space.

`--noOutput` - instructs CC3D not to store any output. This option makes little sense in most cases.

`-h, --help` - prints command line usage on the screen

Example command may look as follows(windows):

```
runScript.bat -i Demos\CompuCellPythonTutorial\InfoPrinter\cellsort_2D_info_printer.
↪cc3d -f 10 -o Demos\CompuCellPythonTutorial\InfoPrinter\screenshots -c infoPrinter
```

linux:

```
runScript.sh -i Demos/CompuCellPythonTutorial/InfoPrinter/cellsort_2D_info_printer.
↪cc3d -f 10 -o Demos/CompuCellPythonTutorial/InfoPrinter/screenshots -c infoPrinter
```

osx:

```
runScript.command -i Demos/CompuCellPythonTutorial/InfoPrinter/cellsort_2D_info_
↪printer.cc3d -f 10 -o Demos/CompuCellPythonTutorial/InfoPrinter/screenshots -c
↪infoPrinter
```

8.7 Managing CompuCell3D simulations (CC3D project files)

Until version `3.6.0` CompuCell3D simulations were stored as a combination of Python, CC3DML (XML), and PIF files. Starting with version `3.6.0` we introduced new way of managing CC3D simulations by enforcing that a single CC3D simulation is stored in a folder containing `.cc3d` project file describing simulation resources (.cc3d is in fact XML), such as CC3DML configuration file, Python scripts, PIF files, Concentration files *etc...* * and a directory called `Simulation` where all the resources reside. The structure of the new-style CC3D simulation is presented in the diagram below:

->CellsortDemo

> CellsortDemo.cc3d

> **->Simulation**

>> Cellsort.xml

>> Cellsort.py

>> CellsortSteppables.py

>> Cellsort.piff

>> FGF.txt

Bold fonts denote folders. The benefit of using CC3D project files instead of loosely related files are as follows:

1) Previously users had to guess which file needs to be open in CC3D – CC3DML or Python. While in a well written simulation one can link the files together in a way that when user opens either one the simulation would work but, nevertheless, such approach was clumsy and unreliable. Starting with `3.6.0` users open `.cc3d` file and they don't have to stress out that CompUCell3D will complain with error message.

> **Warning:** The only way to load simulation in CompuCell3D is to use `.cc3d` project. We no longer support previous ways of opening simulations

2) All the files specified in the .cc3d project files are copied to the result output directory along with simulation results (uncles you explicitly specify otherwise). Thus, when you run multiple simulations each one with different parameters, the copies of all CC3DML and Python files are stored eliminating guessing which parameters were associated with particular simulations.

3) All file paths appearing in the simulation files are relative paths with respect to main simulation folder. This makes simulations portable because all simulation resources are contained withing single folder. In the example above when referring to `Cellsort.piff` file from `Cellsort.xml` you use `Simulation/Cellsort.piff`. This makes simulations easily exchangeable between collaborators

4) New style of storing CC3D simulations has also another advantage – it makes graphical management of simulation content and simulation generation very easy. As a matetr of fact new component of CC3D suite – Twedit++ - CC3D edition has a graphical tool that allows for easy project file management and it also has new simulation wizard which allows users to build template of CC3D simulation within less than a minute.

Let's now look in detail at the structure of `.cc3d` files (we are using XML syntax here):

```xml
<Simulation version="3.6.0">
    <XMLScript>Simulation/Cellsort.xml</XMLScript>
    <PythonScript>Simulation/Cellsort.py</PythonScript>
    <Resource Type="Python">Simulation/CellsortSteppables.py</Resource>
   <PIFFile>Simulation/Cellsort.piff</PIFFile>
    <Resource Type="Field" Copy="No">Simulation/FGF.txt</Resource>
</Simulation>
```

As you can see the structure of the file is quite flat. All that we are storing there is names of files that are used in the simulation. Two files have special tags <XMLFile> which specifies name of the CC3DML file storing "CC3DML portion" of the simulation and <PythonScript> which specifies main Python script. We have also <PIFFile> tag which is used to designate PIF files. All other files used in the simulation are referred to as Resources. For example Python steppable file is a resource of type "Python" - <Resource Type="Python">Simulation/CellsortSteppables.py</Resource> . FGF.txt is a resource of type "Field" - <Resource Type="Field" Copy="No">Simulation/FGF.txt</Resource>. Notice that all the files are specified using paths relative to main simulation directory i.e. w.r.t to the dir in which .cc3d file resides

As we mentioned before, when you run .cc3d simulation all the files listed in the project file are copied to result folder. If for some reason you want to avoid coping of some of the files, simply add Copy="No" attribute in the tag with file name specification for example"

```
<PIFFile Copy="No">Simulation/Cellsort.piff</PIFFile>
<Resource Type="Field" Copy="No">Simulation/FGF.txt</Resource>
```

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